BOOK OF ABSTRACTS

ISAS 2018

International Symposium on Aggregation and Structures

Valladolid (Spain), July 2-5, 2018

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Book of abstracts

Editors: José Luis García-Lapresta, Miguel Martínez-Panero and David Pérez-Román

Cover illustration: Architectonic detail of *Rector Tejerina* building, University of Valladolid. Photography by Juan Carlos Barrena.

Preface

The goal of ISAS 2018 is to give the opportunity to researchers to present and discuss their latest results about aggregation and structures, and to identify new trends in this field. The topic is to be understood in a wide sense: aggregation of structures and aggregation on structures.

The symposium is a follow up of ABLAT 2014 (Trabzon, Turkey), and of ISAS 2016 (Luxembourg City, Luxembourg). During four days of the symposium (July 2-5, 2018), four invited speakers are presenting the state-of-the-art in some particular topics related to lattice-based aggregation. Namely, Jean-Luc Marichal from Luxembourg is opening the symposium presenting Associative and Quasitrivial Operations on Finite Sets: Characterizations and Enumeration. The next invited speaker Salvatore Greco from Catania, Italy and Portsmouth, United Kingdom, is presenting The Bipolar Pawlak-Brouwer-Zadeh Lattice as a Grammar for Reasoning on Data in Multiple Criteria Decision Aiding Using Dominance-Based Rough Set Approach. The third invited speaker Irina Perfilieva from Ostrava, Czech Republic, discusses Nonlocal Operators for Dimensionality Reduction and Image Inpainting. Finally, Manuel Úbeda-Flores from Almería, Spain, is dealing with Constructing Copulas with Given Diagonal and Opposite Diagonal Sections.

From several submissions, after a carefull reviewing, the program committee has chosen 23 contributions for oral presentations. The prepared program allows to follow all presentations and offers an opportunity for a fruitfull discussions among the participants. We are gratefull to the organizing team from the Valladolid University, in particular to José Luis García-Lapresta, for preparing this nice event contributing to the development of the aggregation theory. Note also that this event is an activity of the EUSFLAT working group AGOP, and that the next related event AGOP 2019 will be organized in Olomouc, Czech Republic, in July 2019.

July 2018

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Esteban Induráin, Universidad Pública de Navarra, Spain
Radko Mesiar, Slovak University of Technology, Slovak Republic

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ISAS 2018 is organized by PRESAD Research Group of the University of Valladolid, Spain.

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Contents

Invited Speakers

The Bipolar Pawlak-Brouwer-Zadeh Lattice as a Grammar for Reasoning on Data in Multiple Criteria Decision Aiding Using Dominance-Based Rough Set Approach

Salvatore Greco

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The Dominance-based Rough Set Approach (DRSA) (Greco et al. [6]) extends the classical Rough Set Theory proposed by Zdzis?aw Pawlak (Pawlak [8,9]) in order to handle data expressed on ordered domains. DRSA has proved to be a powerful model in decision aiding, permitting to handle preference information supplied by the decision maker in purely qualitative and ordinal terms. It has been successfully applied in many domains ranging from finance (Greco et al. [5]) to environmental management (Boggia et al. [1]). In this talk we present an algebraic model for DRSA that extends the Brouwer-Zadeh lattice (Cattaneo [2], Cattaneo and Ciucci [3]) and the Pawlak-Brouwer-Zadeh lattice (Greco et al. [7]) introduced for the classical indiscernibility-based rough set approach. The new model permits to distinguish between two kinds of "imperfect" information in case of ordered data, permitting a joint consideration of vagueness due to imprecision typical of fuzzy sets, and ambiguity due to coarseness typical of rough set theory. More precisely, in the context of DRSA applied to ordinal classification with monotonicity constraints, vagueness is due to imprecision in object classification - it appears when an expert is hesitant when classifying the objects because her knowledge of the objects is not perfectly precise; ambiguity is due to coarseness or granularity of the description of the objects by the attributes - it appears when some attribute is missing in the description, or when the considered attributes do not have sufficiently fine scales to avoid violation of indiscernibility or dominance principle. Joint consideration of vagueness and ambiguity within DRSA shows once again a complementary character of fuzzy sets and rough sets in dealing with distinct facets of imperfect knowledge (Dubois and Prade [4]). To build the model we use the bipolar Brouwer-Zadeh lattice to represent a basic vagueness, and to introduce dominance-based rough approximation we define a new operator, called bipolar Pawlak operator. The new model we obtain in this way is called bipolar Pawlak-Brouwer-Zadeh lattice. Using a didactic example, we will show how the logic structure of the Pawlak-Brouwer-Zadeh lattice can be used to aggregate evaluations of multiple experts taking into account the available information.

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Associative and Quasitrivial Operations on Finite Sets: Characterizations and Enumeration

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We investigate the class of binary associative and quasitrivial operations on a given finite set. Here the quasitriviality property (also known as conservativeness) means that the operation always outputs one of its input values. We also examine the special situations where the operations are commutative and nondecreasing, in which cases the operations reduce to discrete uninorms (which are discrete fuzzy connectives playing an important role in fuzzy logic).

Interestingly, associative and quasitrivial operations that are nondecreasing are characterized in terms of total and weak orderings through the so-called single-peakedness property introduced in social choice theory by Duncan Black.

We also address and solve a number of enumeration issues: we count the number of binary associative and quasitrivial operations on a given finite set as well as the number of those operations that are commutative and/or nondecreasing.

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Nonlocal Operators for Dimensionality Reduction and Image Inpainting

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In this talk, we are focused on mathematics that is used for efficient representation of a large data set. Using the notion of the Laplacian of a graph, we consider a low-dimensional representation of the data set that optimally preserves local neighborhood information. The representation map may be viewed as a discrete approximation to a continuous map that naturally arises from the geometry of the underlying manifold. Classical approaches to dimensionality reduction include principal components analysis (PCA) and multidimensional scaling.

In the fuzzy literature, the dimensionality reduction is hidden under the notions and techniques of granulation, clustering and fuzzy partition. The results are used in the form of collections of fuzzy sets and after that in fuzzy rule databases. Actually, the main advantage of modeling with fuzzy IF-THEN rules is in transforming a problem from an initial complex high dimensional data space to the low dimensional space where fuzzy sets are new atomic units.

However, despite of this obvious similarity, the dimensionality reduction in the sense of machine learning is different. The essential difference is in the way of representation. Instead of center-shape (clustering) or membership function (fuzzy sets) representation, low-dimensional images are characterized in terms of features (eigenvectors). Therefore, a cluster (granule) is characterized as a collection of common features that are extracted from an initial data embedded into a particular manifold. In this respect, the only fuzzy technique which is similar to the machine-learning-based dimensionality reduction is the F-(fuzzy) transform.

Functional F-transform components are solutions to a certain specification of the dimensionality reduction problem. On the other hand, the F-transform components closely correspond to nonlocal operators. Nonlocal operators appear naturally in a wide range of applications, e.g., in the investigation of gravitational or acoustic fields, image and signal processing. "Non-locality" means that any point can interact directly with any other point in the image domain. Nonlocal operators will be introduced using the gradient and divergence definitions on graphs, because the graph representation is naturally associated with modeling neighborhood relationships between the data elements.

In the talk, we establish a connection between a space with a fuzzy partition and its graph representation, F-transform components and nonlocal operators. In the application part, we consider the problem of image restoration and explain how it can be properly formulated and solved in the language of F-transform and using the technique of dimensionality reduction.

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Constructing Copulas with Given Diagonal and Opposite Diagonal Sections

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Copulas, multivariate probability distribution functions with uniform univariate margins on [0,1], are a special type of conjunctive aggregation functions. Methods to construct copulas with some partial information have been proposed in the literature. In this talk we review various methods for constructing bivariate (and multivariate) copulas with given diagonal sections, from seminal works to the most recent research on copulas with given diagonal and opposite diagonal sections. A generalization of copulas with given diagonal plane sections in higher dimensions and other sections that generalize the diagonal and opposite diagonal sections are of particular interest.

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Contributed Talks

First-Order Then-Aggregate Strategies to Solve the Optimal Bucket Order Problem

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The *optimal bucket order problem (OBOP)* is an aggregation distance-based problem which consists in obtaining a consensus partial ranking from a matrix of preferences/precedences. In this work we present a family of *first-order then-aggregate* algorithms to deal with the OBOP. We carry out a wide experimental study which shows not only the significant performance of the algorithms regarding accuracy, but also their scalability to handle high dimensional problem instances.

1 Introduction

The term *Rank aggregation* shelters a series of problems whose aim is to provide a consensus ranking from a dataset containing preference information about a set of items [1]. Currently, rank aggregation is a very active field of research where many disciplines converge: social sciences, mathematics and computer sciences, among others.

In this work we focus on the *optimal bucket order problem* (OBOP), a distance-based optimization problem introduced in [2]. In the OBOP, the input is a precedence matrix which usually condenses the preferences regarding a set of items expressed in a dataset. Then, the objective is to obtain a complete ranking with ties, or equivalently, a *bucket order matrix*, that minimizes the L^1 matrix distance with respect to the input precedence matrix.

Since the OBOP is NP-hard [2], greedy heuristics are normally used to solve it. In this sense, the bucket pivot algorithm (BPA) [2,3] is the standard option, as long as some recently-presented improved BPA-methods [4]. In this setting, LIA_G^{MP2} can be considered the current state-of-the-art algorithm. Also evolution strategies have been recently used to tackle the OBOP [5].

In [6,7] some *Cluster-first sort-second* strategies are considered to deal with the OBOP. These approaches are based on the idea of first clustering the items that are similar in terms of their precedences (i.e. constructing buckets), and then sorting these buckets. Thus, while the pivot based strategies consider both

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tasks simultaneously (ordering and clustering), this approach decouples these steps. The algorithms SortCC and PivotCC may be considered as the paradigm in this setting.

In this work, we propose *first-order then-aggregate* (FOTA) strategies to solve the OBOP. They consists in first sorting the items, and then clustering them in buckets suitably in order to minimize the objective function. In our proposal, the first phase (namely, sorting the items) is based on a Borda-inspired algorithm [8].

We carry out a wide experimental study which shows not only the significant performance of the FOTA algorithms regarding accuracy, but also their scalability to handle high dimensional problem instances.

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Extending Aggregation Functions for Undefined Inputs

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In practice, some inputs of aggregation functions [3] may happen to be undefined. We introduce several systematic ways of treating missing arguments in aggregation functions, inspired by a similar treatment of missing degrees in fuzzy partial logic [2,1]. Let us illustrate the problem on a toy example.

Example. Consider a teacher who wants to aggregate each student's results from several tests, graded on a numerical scale (say, 0–100 %). Depending on the teacher's policy, tests missed by a student can be:

(a) Ignored, i.e., the final grade is calculated just from the tests actually taken.
(b) Regarded as failed, i.e., the grade of 0 % is assigned to all missed tests.
(c) Strictly required for assigning the final grade, so the students who have not taken all of the tests cannot yet obtain the final grade.
(d) Treated as in (c), but if the student has failed in one of the tests taken, the final grade will be *Fail* in any case (i.e., passing all taken tests is required for passing the final exam); etc.

Remark. In the setting of the Example, a rational policy should distinguish between valid and invalid excuses for missing a test—e.g., using (a) in the case of illness and (b) otherwise; for simplicity, though, we only consider missing values of a single type and leave the treatment of multiple types of undefined data for future work.

In order to handle undefined inputs in aggregation, we equip the set $\mathbb{R}$ of reals (or generally, any set X of values) with an extra element $*$, intended to represent missing data and to be understood as a *NaN* ('not a number') value added to $\mathbb{R}$. Given an aggregation function on $\mathbb{R}$, we extend it to $\mathbb{R}_* = \mathbb{R} \cup \{*\}$ in several uniform ways, so that the extended function can accept $*$ as input or output values. In the setting of the Example, the policies (a)–(d) respectively correspond to the *Sobociński, 0-fill, Bochvar,* and *Kleene* extensions of the aggregation function. (The names come from the corresponding three-valued logics; there are further meaningful extensions omitted here for simplicity.) In the definition below, we treat both finitary and infinitary aggregation functions jointly.

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Definition. Let an operator $\bigoplus = \bigcup_{I \in \mathcal{J}} \bigoplus_I$ be given, where $\mathcal{J}$ is a class of index sets I and $\bigoplus_I \colon X^I \to X$, for a fixed set X of values (often, $X \subseteq \mathbb{R}$). If $\bar{x} \in X^I$, we also write $\bigoplus_I(\bar{x})$ as $\bigoplus_{i \in I} x_i$. Let furthermore $X_* = X \cup \{*\}$, where $* \notin X$. Then we define the following X_*-extensions of $\bigoplus$:

– The *Bochvar* extension $\bigoplus^{\mathrm{B}}_{i \in I} x_i = \begin{cases} \bigoplus_{i \in I} x_i & \text{if } (\forall i \in I)(x_i \neq *) \\ * & \text{otherwise.} \end{cases}$

– The *Sobociński* extension $\bigoplus^{\mathrm{S}}_{i \in I} x_i = \begin{cases} \bigoplus_{i \in I, x_i \neq *} x_i & \text{if } (\exists i \in I)(x_i \neq *) \\ * & \text{otherwise.} \end{cases}$

– The *Kleene* extension $\bigoplus^{\mathrm{K}}_{i \in I} x_i = \begin{cases} o & \text{if } (\exists i \in I)(x_i = o) \\ \bigoplus^{\mathrm{B}}_{i \in I} x_i & \text{otherwise,} \end{cases}$
  if o is the absorbing element of $\bigoplus$, i.e., if $\bigoplus_{i \in I} x_i = o$ whenever $x_i = o$ for some $i \in I$.

– The $\bar{c}$-*fill* extension $\bigoplus^{\mathrm{F}_{\bar{c}}}_{i \in I} x_i = \bigoplus_{i \in I} \mathrm{F}_{\bar{c}}(x_i)$, where $\mathrm{F}_{\bar{c}}(x_i) = \begin{cases} x_i & \text{if } x_i \neq * \\ c_i & \text{if } x_i = *, \end{cases}$
  for $\bar{c} \in X^I$. (Often, $\bar{c}$ is a constant function, i.e., $\bar{c} \colon I \to \{a\}$ for $a \in X$.)

Examples. Useful extensions of well-known operators and aggregation functions on $X \subseteq \mathbb{R}$ include, e.g., those of:

– Suprema and infima ($\bigvee^{\mathrm{B}}$, $\bigwedge^{\mathrm{S}}$, $\bigvee^{\mathrm{K}}$, etc.)
– Arithmetic, geometric, or harmonic means (A^{S}, G^{K}, H^{B}, etc.), OWA, etc.
– Sums, products, integrals ($\sum^{\mathrm{B}}, \prod^{\mathrm{K}}, \int^{\mathrm{S}}$, etc.), mean values, etc.

Various properties of aggregation functions (e.g., monotony, boundedness, associativity, etc.) transfer to their X_*-extensions (in an appropriately modified form). We will present samples of such results, both on particular functions (for instance, the complete distributivity of $\bigvee^{\mathrm{S}}$ over $\bigwedge^{\mathrm{B}}$) and on general properties (such as the preservation of associativity in Bochvar and Sobociński extensions, modified min/max bounds of X_*-extensions of aggregation functions w.r.t. three prominent orders on X_*, etc.). We will also tout several benefits of extending functions to X_* (such as the relaxation of definedness conditions, preservation of covariant quantities, etc.) and discuss the applicability of the introduced notions in several application areas.

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On a New Construction Method of Fuzzy Sheffer Stroke Operation

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In classical logic, Sheffer stroke is one of the two operations that can be used by itself, without any other logical operations, to constitute a logical formal system. Despite its importance, this logical connective has only been recently introduced in the fuzzy logic framework [2,1]. In particular, in [1], the authors introduced fuzzy Sheffer stroke as follows.

Definition 1 (cf. [1, Definition 2.5]). *A function $D\colon [0,1]^2 \to [0,1]$ is called a **fuzzy Sheffer stroke operation** (or fuzzy Sheffer stroke) if it satisfies, for all $x, y, z \in [0,1]$, the following conditions:*

(D1) $D(x,z) \geq D(y,z)$ *for* $x \leq y$, *i.e.,* $D(\cdot, z)$ *is non-increasing,*
(D2) $D(x,y) \geq D(x,z)$ *for* $y \leq z$, *i.e.,* $D(x,\cdot)$ *is non-increasing,*
(D3) $D(0,1) = D(1,0) = 1$ *and* $D(1,1) = 0$.

Several examples were presented and some preliminary results related to the construction of other fuzzy logic connectives from fuzzy Sheffer strokes were studied. Especially important is the characterization of these operators the negation of a conjunction as the following result shows.

Theorem 1 (cf. [1, Theorem 3.1]). *Let $D\colon [0,1]^2 \to [0,1]$ be a binary operation. Then the following statements are equivalent:*

(i) D is a fuzzy Sheffer stroke.
(ii) There exist a fuzzy conjunction C and a strict fuzzy negation N such that $D(x,y) = N(C(x,y))$ for all $x, y \in [0,1]$.

Moreover, in this case, $C(x,y) = N^{-1}(D(x,y))$ for all $x, y \in [0,1]$.

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The previous theorem provides a straightforward way to generate fuzzy Sheffer strokes from a fuzzy conjunction and a fuzzy negation. However, this construction method is not particularly efficient to study the additional properties, desirable for a particular application, of these operators since it is difficult to relate the additional properties of the fuzzy conjunction to the ones of the fuzzy Sheffer stroke through a fuzzy negation. Therefore, the goal of this paper is to propose a direct construction method of fuzzy Sheffer strokes through the use of two univalued functions.

Definition 2. *Let $f : [0,1] \to [0,+\infty]$ be a decreasing function with $f(0) = +\infty$ and $f(1) = 0$, and let $g : [0,+\infty] \to [0,1]$ be an increasing function with $g(0) = 0$ and $g(+\infty) = 1$. The operator $D_{f,g} : [0,1]^2 \to [0,1]$ defined by*

$$D_{f,g}(x,y) = g(f(x) + f(y))$$

is called an (f,g)-Sheffer stroke. In this case, the pair of functions (f,g) is called a pair of additive generators of $D_{f,g}$.

Note that the additive generators of $D_{f,g}$ can be non-continuous functions. In particular, the maximum fuzzy Sheffer stroke $D_{\max}$ given by

$$D_{\max}(x,y) := \begin{cases} 0, & \text{if } (x,y) = (1,1), \\ 1, & \text{otherwise,} \end{cases}$$

is an (f,g)-Sheffer stroke generated from non-continuous additive generators.

It can be proved that some operators of this family have a strong relationship with strict Archimedean t-norms. In fact, when a strict fuzzy negation N and a continuous and strictly increasing function g are considered, then $N^{-1} \circ D_{f,g}$ is a t-norm (which turns out to be strict Archimedean) if and only if $f = g^{-1} \circ N$. In this case, the following interesting subfamily arises

$$D_{g,N}(x,y) = g\left(g^{-1}(N(x)) + g^{-1}(N(y))\right), \qquad x,y \in [0,1].$$

Several of the additional properties introduced in [1] are studied for this family concluding that they are symmetric, they satisfy $D(D(x,x),D(x,x)) = x$ in some cases but they never satisfy $D(1,x) = D(x,x)$ for all $x \in [0,1]$.

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Aggregation vs. Disaggregation: A geometrical approach

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In the fuzzy set theory we find aggregation problems: starting from fuzzy subsets we can construct a new one following some criteria. If we consider fuzzy sets on a finite universe with n elements, those fuzzy sets can be considered as vectors in the n-dimensional unit cube, and thus we can consider aggregation operators based on geometrical methods. For example we could look for Fermat and Weber points.

Another geometrically based method could be inspired in experimental scientists behavior analyzing graphical data in euclidean spaces. Various repetitions of a single measure are expected to be close to each other, and at first sight they are detected when various anomalies in measurement occurs (called outliers). Generally outliers have to be removed from analysis because they lead to wrong results. The aggregation process proposed identifies points with more data in their neighborhood by taking into account some kind of distance obtained maximizing a quotient of "points in neighborhood" per "neighborhood size" (which captures the initial idea). This aggregation has interesting attributes as ignoring outliers or distinguishing when source of data are different enough.

We could consider another geometrical method inspired in the non-parametric asset allocation model denoted Entropy Pooling introduced by Meucci [10], which combines an arbitrary market model with completely general views on this market, in order to produce a a posterior distribution. For this, the views are interpreted as statements that deform the prior distribution so that the minimum of unnecessary structure is imposed, measuring this discrepancy through the entropy. If there are more than one investor expressing views, with different levels of confidence, we can aggregate all of them using the opinion pooling technique. Based on this idea, we could reinterpret it from a geometrical point of view giving us an aggregation method for fuzzy sets on a finite universe, that can be considered as vectors in the n-dimensional unit cube, and so they can play the role of vectors of probabilities representing the market model. Considering the views, which can be easily translated into geometrical conditions in the unit cube, we could combine them through the entropy pooling technique in order to obtain a new vector in the n-dimensional unit cube and so a new finite fuzzy set.

Considering the last method of aggregation, it inspires us the inverse problem, which may be little known. That is, given the result of a fuzzy aggregation and knowing that such an aggregation has been obtained following certain criteria, it is about trying to obtain information about which were the original sets that gave rise to such aggregation. Roughly speaking, we will call this concept disaggregation.

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A General Approach to Aggregation Operators

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The aim of this paper is to propose a general unified framework for defining aggregation operators. Our framework is abstract and algebraic in nature and in this framework we generalize some results in [2], [3], [5] and [12].

We consider convex structures where the notion considered here (see [13]) is not restricted to the context of vector spaces. The basic idea of our approach is to describe the space of alternatives in terms of a *betweeness* relation. We can prove that lattices, median spaces and interval spaces are convex spaces and also that to every property spaces (see [2], [3] and [12]) is associated a convex structure.

We then focus on aggregation operators $f: X^A \to X$ where X is convexity space and A is a nonempty set. We consider operators that are component-wise compatible with the structure of convexity space of X. Moreover we study compatible aggregation operators that satisfy properties of monotonicity and independence and we consider aggregation operators that are based on *decisive* subsets of A.

We propose also a particular version of Arrow's theorem thus considering a link between aggregation theory and social choice theory.

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Some Results on Internal and Locally Internal Uninorms on Bounded Lattices

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Uninorms as a generalization of triangular norms (t-norms, for short) and triangular conorms (t-conorms, for short) leave the freedom for the neutral element to be an arbitrary element from a bounded lattice rather than at one or zero in that case of t-norms and t-conorms. In this contribution, we introduce the concept of internal uninorm on an arbitrary bounded lattice L. We investigate some properties of these operators and the relationship of them with locally internal uninorms. We show that an internal uninorm need not always exist on an arbitrary bounded lattice. Furthermore, based on the Zermelo's well-ordering theorem, we propose two construction methods to obtain internal uninorms on a bounded lattice L with some additional constraints.

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F-transform with Undefined Inputs

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Undefined real values, as a source of various bugs, are present in everyday practise and can be treated similarly as undefined truth values in a partial fuzzy logic [2,1]. The connectives of a partial fuzzy logic handle undefined truth values represented by a dummy value $*$ that stands aside the scale for truth values L. Extensions to undefined truth values that was applied to connectives of fuzzy logic can be carried for an arbitrary binary function in the following manner.

Definition 1. *Let $X \neq \emptyset$, $* \notin X$, $a \in X$, and $o\colon X^2 \to X$. We define operations o_{B} and o_{S}, moreover, if a is an absorbing element of o then we define o_{K}, from $(X \cup \{*\})^2$ to $X \cup \{*\}$ as follows:*

o_{B}	y	$*$
x	$o(x,y)$	$*$
$*$	$*$	$*$

o_{S}	y	$*$
x	$o(x,y)$	x
$*$	y	$*$

o_{K}	a	y	$*$
a	a	a	a
x	a	$o(x,y)$	$*$
$*$	a	$*$	$*$

(1)

where $x, y \neq *$ in the case of $o_{\mathrm{B}}, o_{\mathrm{S}}$ and $x, y \notin \{a, *\}$ in the case of o_{K}.

We call o_{B} the Bochvar-extension *of o, o_{S} the* Sobociński-extension *of o and o_{K} the* Kleene-extension *of o to undefined value represented by $*$.*

- $*$ represents not a number NaN.
- These extensions are motivated by classical three valued logics.
- A generalization of extensions to n-ary functions is straightforward.
- They are applicable to aggregation operators.
- Our choice of an extension should be due to a required behaviour.

Let us demonstrate how the above introduced extensions work on Fuzzy Transform technique that use weighted arithmetic mean to compute transformation components. Recall a discrete fuzzy transform from [3], where $X \neq \emptyset \subset \mathbb{R}$ and a continuous function $f\colon X \to \mathbb{R}$ is given only at some non-empty finite set of points $D \neq \emptyset \subset X$. Select basic functions $A_1, \ldots, A_n$ (where $A_i\colon X \to [0,1]$ and fulfill some additional requirements) and define a (direct discrete) *F-transform* of f as a vector $F_n[f] = (F_1, \ldots, F_n)$, where the k-th *component* F_k is equal to

$$F_k = \sum_{d \in D} f(d) A_k(d) \Big/ \sum_{d \in D} A_k(d), \quad k = 1, \ldots, n.$$

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The *inverse discrete F-transform* of f with respect to $(F_1, \ldots, F_n)$ and $A_1, \ldots, A_n$ is a function $f_{F,n} \colon X \to \mathbb{R}$ such that

$$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x), \quad x \in X.$$

Indeed, f is total on D and partial on X. Therefore, it can be considered undefined on $X \setminus D$. This fact can be formalized by extension of real-line by a dummy element $*$ (represents undefinability) to $\mathbb{R}_* = \mathbb{R} \cup \{*\}$ and f to $f^* \colon X \to \mathbb{R}_*$ as follows:

$$f^*(x) = \begin{cases} f(x), & x \in D; \\ *, & \text{otherwise.} \end{cases}$$

Now, we can introduce F*-transform components for f^* as

$$F_k^* = \sum_{x \in X}{}_{\mathrm{S}} (f^*(x) \cdot_{\mathrm{B}} A_k(x)) /_{\mathrm{B}} \sum_{x \in X} (A_k(x) \cdot \chi_D(x)), \quad k = 1, \ldots, n,$$

where χ_D denotes characteristic function of D.

The *inverse discrete F*-transform* of f^* with respect to $(F_1^*, \ldots, F_n^*)$ and $A_1, \ldots, A_n$ is a function $f_{F,n}^* \colon X \to \mathbb{R}_*$ such that

$$f_{F,n}^*(x) = \sum_{k=1}^{n}{}_{\mathrm{B}} (F_k^* \cdot_{\mathrm{K}} A_k(x)).$$

This definition of F*-transform that operates on $*$-extended reals allows us to fill in "small" gaps in the given data D by means of real values given by inverse F*-transform while "big" gaps remain undefined, i.e., inverse F*-transform takes value $*$ there. Further, we will study analogous properties as given in [3].

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On the Definition of a Penalty Function in and beyond the Framework of Real Numbers

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A common method for the aggregation of real numbers is based on the minimization of a so-called penalty function [1]. Formally, a penalty function is a function $P : [a, b] \times [a, b]^n \to \mathbb{R}$ (for a closed and bounded subinterval $[a, b]$ of the real line) where $P(y; \mathbf{x})$ represents the disagreement of a consensus element y with a list of elements $\mathbf{x}$. In a penalty-based aggregation problem, the aggregate of a list of elements $\mathbf{x}$ is then considered to be (one among) the minimizer(s) of $P(\cdot; \mathbf{x})$. Typical examples of penalty-based aggregation functions are the median (with $P(y; \mathbf{x}) = \sum_{i=1}^{n} |x_i - y|$) and the mean (with $P(y; \mathbf{x}) = \sum_{i=1}^{n} (x_i - y)^2$). If minimal conditions are imposed on the penalty function ((i) $P(y; \mathbf{x}) \geq 0$ and (ii) $P(y; \mathbf{x}) = 0$ if and only if $\mathbf{x} = (y, \ldots, y)$), penalty-based aggregation amounts to idempotent aggregation. However, in order to add some desirable semantics, additional conditions (e.g., (iii) quasi-convexity and lower semi-continuity for a fixed $\mathbf{x}$; or (iii') $P(y; \mathbf{x}) \leq P(y; \mathbf{x}')$ if $x_i' \leq x_i \leq y$ or $y \leq x_i \leq x_i'$ for any i) have been imposed on the penalty function [2].

Outside the framework of real numbers, aggregation on many different structures has been performed in a similar manner without a proper definition of a penalty function. For instance, the method of Kemeny [3] for the aggregation of rankings is based on the penalty $P(y; \mathbf{x}) = \sum_{i=1}^{n} K(y, x_i)$, where K denotes the Kendall distance function between rankings [4]. Similarly, the median procedure for the aggregation of binary relations [5,6] is defined by the penalty $P(y; \mathbf{x}) = \sum_{i=1}^{n} \delta(y, x_i)$, where δ denotes the symmetric difference distance function between binary relations. For the aggregation of relations, the closest strings [7] are those that minimize the penalty $P(y; \mathbf{x}) = \max_{i=1}^{n} H(y, x_i)$, where H denotes the Hamming distance function between strings (of the same length) [8], and median strings [9] are those that minimize the penalty $P(y; \mathbf{x}) = \sum_{i=1}^{n} L(y, x_i)$, where L denotes the Levenshtein distance function between strings [10].

In this presentation, we will discuss how, when moving from real numbers to other structures, properties of type (i), (ii) and (iii') naturally appear in the associated penalty functions, whereas this is not the case for properties of type (iii) [11]. This is similar to the case of the mode for the aggregation of real numbers, which satisfies properties (i), (ii) and (iii'), but fails to satisfy property (iii).

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On Associative, Idempotent, Symmetric, and Nondecreasing Operations

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The study of aggregation functions defined on finite ordinal scales (i.e., finite chains) encounters an increasing interest since the last decades (see, e.g., [1,2,3,5,7,9,10,11,12,13,14,15,16]). Among these functions, discrete t-norms, t-conorms, uninorms, and nullnorms are binary operations that play an important role in fuzzy logic. In particular, these operations share the properties of being associative, symmetric, and nondecreasing (in each variable).

It is known that the class of associative, idempotent, and symmetric binary operations is in one-to-one correspondence with the class of partial orders of semilattices (see, e.g., [6]). We provide a full description of the class of associative, idempotent, symmetric, and nondecreasing binary operations defined on a finite chain in terms of properties of the Hasse diagram of the corresponding semilattice. In particular, given an operation belonging to the latter class, we provide a recursive construction of the corresponding semilattice. We also provide an associativity test for idempotent, symmetric, and nondecreasing operations. Moreover, the enumeration of the class of associative, idempotent, symmetric, and nondecreasing operations leads to a new occurrence of the Catalan numbers and provides another construction of this sequence.

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A Mathematical Study of the Order Structure of Distributed Systems

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In the present talk we propose a mathematical definition of a 'distributed system'. Through this new definition we achieve a mathematical structure made by means of orderings: the processes are represented through total preorders and the communications are characterized by means of biorders. This new definition let us compare some problems related to this field (computer science) with some others related to economics and decision theory, in particular, questions associated to the numerical representability of the order structure.

The concept of 'near-finite partial orders' is introduced as a finite family of chains with a finite communication between them. The representability of this kind of structure is studied, achieving a construction method for a Richter-Peleg multi-utility representation by means of utilities and random structures.

Some other aspects and techniques for the study of distributed systems are also included, in particular, how to aggregate new processes to distributed systems with some kind of network topologies.

1 Introduction

In this work we use the concept of biorder in order to mathematically formalize the order structure of a distributed system. This study was started in [5] for the particular case of distributed systems of two processes.

Since each process consists of a sequence of events, each process is a totally ordered set, and the communication through messages between the processes can be mathematically formalized by means of biorders.

In order to formalize completely the concept of distributed system, a further study is done, but now linking biorders between n totally preordered sets through different network topologies. The present work tries to provide some mathematical tools to study these structures in computing, physics or decision theory.

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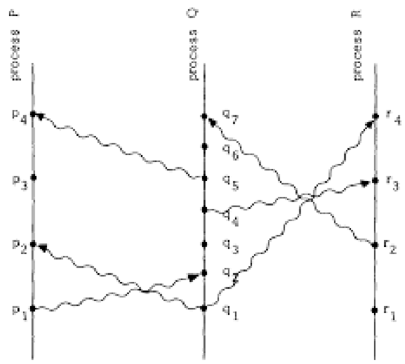

Fig. 1. Illustration of a distributed system with three processes, taken from the paper [9] of Leslie Lamport.

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Poverty Trends in Europe: A Multivariate Dependence Analysis Based on Copulas

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There is a widespread agreement that poverty is a multimensional phenomenon involving not only low incomes, but also deprivations in other dimensions like education, health or labour. In this multidimensional setting, analysing the dependence between dimensions becomes an important issue since higher dependence means higher concentration of deprivations and this could make overall poverty worse; see, for instance, [1], [2] and [3]. In spite of its relevance, the problem of measuring the dependence between dimensions of poverty has been scarcely addressed in the literature and this is the scope of this paper.

We look at multidimensional poverty in Europe by focusing on the AROPE (At Risk Of Poverty or social Exclusion) rate. We select this indicator because it is the headline indicator to monitor and implement effective poverty-reduction policies in the framework of the Europe 2020 Strategy. We propose measuring the multivariate dependence among the three dimensions included in the AROPE rate (income, material needs and work intensity) using copula-based methods.

The copula function is an aggregate function whose arguments come from mapping the original variables into its ranks in the interval [0,1]. Hence, the copula approach focuses on the positions of the individuals across dimensions, rather than on the specific values that those dimensions attain for such individuals. The advantage of this approach is that it enables the decomposition of the joint distribution function of all dimensions into its univariate marginals and the dependence structure captured by the copula. Moreover, copulas allow building scaled-free measures of dependence that capture other types of dependence beyond linear correlation. Furthermore, in a multivariate setting, neither the concept of concordance nor the generalization of the bivariate coefficients of concordance is unique. For instance, in the trivariate case, there are more than eight copula-based generalizations of the well-known bivariate Spearman's rho coefficient. In particular, we apply four of these coefficients, namely those proposed by [5,6], which are based on average orthant dependence concepts, as well as the coefficient proposed by [4] for the trivariate case.

We first review the definitions and main properties of these coefficients. Then, we apply these coefficients to measure how the dependence between the three dimensions included in the AROPE rate has evolved in the 28 EU countries over the period 2008 until 2014. The data we use comes from the EU-Statistics on Income and Living Conditions (EU-SILC) survey, which is the EU reference source for comparative statistics on income distribution and social inclusion at the European level.

Our results show variations between EU countries, but in most of them we observe that, regardless of the coefficient considered, there has been a general increase in the multivariate dependence between poverty dimensions over the period analysed. Noticeably, the highest increase corresponds to Greece and Spain. Moreover, in general, the maximal dependence of poverty is found in the lower orthant over all the years considered. These results suggest that small (high) values of the three poverty dimensions tend to occur together, and this simultaneous concentration of small (large) values of income, no material privations and work intensity, is more likely to occur in 2014 than in 2008. Therefore, it seems that, after the crisis, most EU countries have become more polarized.

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Aggregating Quantitative and Qualitative Assessments in Multidimensional Welfare

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There is considerable agreement that social welfare is a multidimensional concept and several attributes have to be taken into consideration to evaluate the actual level of welfare in a society. The literature on this topic has grown considerably (see for instance Chakravarty [1] for a comprehensive survey on the subject). However, most of the papers in this multidimensional approach focus on dimensions that are measured in a ratio-scale, such as income, life expectancy or unemployment rate. In contrast, many relevant welfare attributes cannot be represented in a quantitative scale. Examples are access to a number of goods or services, such as water, electricity or the Internet, that are represented by dichotomous variables. And still others, such as health status, happiness, personal security, environmental quality, or level of education, that are usually measured in ordered qualitative scales. In this contribution we consider a two-step procedure for aggregate dimensions of well-being regardless of the nature of the attribute. Once each dimension is normalized (the normalization procedure depends on the kind of attribute), we first aggregate across dimensions for each individual and, subsequently, we aggregate across individuals.

Ordinal proximity measures. The notion of ordinal proximity measure was introduced by García-Lapresta and Pérez-Román [2]. Consider an ordered qualitative scale (OQS) $\mathcal{L} = \{l_1, \dots, l_g\}$ such that $l_1 < \cdots < l_g$ and $g \geq 3$, whose elements are linguistic terms, and a linear order $\mathit{\Delta} = \{\delta_1, \dots, \delta_h\}$, with $\delta_1 \succ \cdots \succ \delta_h$. The elements of $\mathit{\Delta}$ are not numbers, but different degrees of proximity, being δ_1 and δ_h the maximum and minimum degrees, respectively.

Definition ([2]). An *ordinal proximity measure (OPM) on* $\mathcal{L}$ *with values in* $\mathit{\Delta}$ is a mapping $\pi : \mathcal{L}^2 \longrightarrow \mathit{\Delta}$, where $\pi(l_r, l_s) = \pi_{rs}$ means the degree of proximity between l_r and l_s, satisfying the following conditions:

1. *Exhaustiveness*: For every $\delta \in \mathit{\Delta}$, there exist $l_r, l_s \in \mathcal{L}$ such that $\delta = \pi_{rs}$.
2. *Symmetry*: $\pi_{sr} = \pi_{rs}$, for all $r, s \in \{1, \dots, g\}$.
3. *Maximum proximity*: $\pi_{rs} = \delta_1 \Leftrightarrow r = s$, for all $r, s \in \{1, \dots, g\}$.
4. *Monotonicity*: $\pi_{rs} \succ \pi_{rt}$ and $\pi_{st} \succ \pi_{rt}$, for all $r, s, t \in \{1, \dots, g\}$ such that $r < s < t$.

Consider a society composed of n individuals, $I = \{1, \dots, n\}$, is evaluated under q well-being dimensions (attributes or criteria). These dimensions could be measured on quantitative or qualitative scales, depending on their nature.

Quantitative assessments. If dimension $d \in \{1, \ldots, q\}$ is measured in a quantitative scale (e.g. $\{0, 1\}$, an interval $[a, b]$ or $\mathbb{R}$), let $u_d : I \longrightarrow \mathbb{R}$ be the function that assigns the corresponding numerical assessment to each individual $i \in I$ regarding dimension d, $u_d(i) \in \mathbb{R}$.

In order to all the numerical assessments belong to the unit interval, different normalization procedures have been considered in the literature. We consider three different situations.

1. If the dimension is represented by a $\{0, 1\}$-dichotomous variable, no normalization procedure is needed.
2. If the dimension is measured in an interval $[a, b]$ (for instance $[0, 100]$, if the dimension is measured in percentages), then we denote by $m = a$ and $M = b$ the two extremes of the interval.
3. In the rest of the cases, since the population is finite, it is always possible to find two real numbers $m < M$ such that $m \leq \min u_d \leq \max u_d \leq M$.

It is possible to generate an index $u_{id} \in [0, 1]$ that is an affine transformation of $u_d(i)$: $u_{id} = \alpha \cdot u_d(i) + \beta$, with $\alpha > 0$ and $\beta \in \mathbb{R}$. The normalization procedure we propose in cases 2 and 3 is the following:

$$u_{id} = \frac{u_d(i) - m}{M - m}.$$

Qualitative assessments. If dimension $d \in \{1, \ldots, q\}$ is measured with an OQS $\mathcal{L}_d = \{l_1, \ldots, l_{g_d}\}$ equipped with an OPM $\pi_d : (\mathcal{L}_d)^2 \longrightarrow \varDelta_d = \{\delta_1, \ldots, \delta_{h_d}\}$, let $u_d : I \longrightarrow \mathcal{L}_d$ be the mapping that assigns the corresponding qualitative assessment to each individual $i \in I$ regarding dimension d, $u_d(i) \in \mathcal{L}_d$. In order to the qualitative assessment $u_d(i)$ be transformed to a numerical value within the unit interval, we consider

$$u_{id} = \frac{\rho\big(\pi_d(u_d(i), l_1)\big) - \rho\big(\pi_d(u_d(i), l_{g_d})\big) + h_d - 1}{2(h_d - 1)},$$

where $\rho(\delta_r) = r$.

Aggregation. Let $A : [0, 1]^q \to [0, 1]$ and $B : [0, 1]^n \to [0, 1]$ be two aggregation functions. The overall assessment of individual $i \in I$ is $u_i = A(u_{i1}, \ldots, u_{iq})$. In turn, $u_I = B(u_1, \ldots, u_n)$ is the overall assessment of society I (see Seth [3]). Given two societies I and J, they can be ranked through the following weak order on the set of all possible societies: $I \succeq J \Leftrightarrow u_I \geq u_J$.

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A Note on Generating Sets for Aggregation Clones on Finite Lattices

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Aggregation functions on bounded posets are defined as monotone operations fulfilling the boundary conditions. Given a bounded poset $(P, \leq, 0, 1)$, it can be easily seen that any projection operator on P is an aggregation function and that the set of all aggregation functions is closed with respect to composition of functions. Hence, from an algebraic point of view, aggregation functions correspond to 0,1 monotone clone. It is the well-known fact that the aggregation clone on any finite lattice is finitely generated, contrary to the case of finite bounded posets, where this need not be true in general. Some generating sets for the aggregation clone were studied in [2,3], and in [1] for the idempotent aggregation clone respectively.

The aim of this contribution is to provide some general method for finding certain generating set of the aggregation clone. Specifically, we show a sufficient condition for a system of aggregation functions to be included in a generating set containing also the lattice operations and certain unary functions capturing the order structure of the underlying lattice. Based on these results, with respect to constraints imposed on the arities of generating functions, we also present some upper bounds for the minimal cardinality of generating sets.

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Generating Sets of Idempotent Aggregation Clones on Bounded Lattices and their Minimality

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The major contribution in aggregation theory have delt with some real interval scales and they were summarized in many monographs, see e.g. [4].

Only recently more abstract scales were considered, in particular lattice (poset) scales. For information sciences, and in particular for subjective decision problems, typical scales deal with bounded (distributive) lattices. Not going into details, among different papers dealing with aggregation functions acting on lattices, we recall e.g. the seminal papers [2,3], our recent papers [6,7], or the papers on nullnorms and uninorms on bounded lattices etc.

Particular classes of aggregation functions can be seen as special clones, ranging from the smallest one (all projections) to the biggest one (all aggregation functions of a considered lattice L). One special case is related to the class of all idempotent aggregation functions on L, and its subclass of idempotent lattice polynomials (i.e., Sugeno integrals, see [2,3]). Note that in the case of real intervals, idempotent aggregation functions are just monotone means, and they are indispensable in several domains related to unanimous decision making.

Possible complexity of the above mentioned clones can be reduced when we look at their generating sets, i.e., subsets of aggregation functions from the considered clone such that their compositions generate all members of this clone. This problem was discussed for idempotent lattice polynomials on a bounded distributive lattice L in [2,3], see also [5], and for all idempotent aggregation functions on a bounded lattice L in [1]. Note that the generating set introduced in [1] has contained also quite artificial ternary aggregation functions and it was far from to be minimal.

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In out talk:

- we shall present the improvement of our earlier results concerning the generating sets of idempotent aggregation functions on lattices, where only binary generating idempotent aggregation functions will occur
- we shall discuss the minimality problems of introduced generating sets
- we shall present a binary generating set of the class of all Sugeno integrals on L.

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Continuity and Divisibility of Monotone Binary Operations on Bounded Posets

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At the the 3rd International Symposium on Fuzzy Sets – Uncertainty Modelling 2017 in Rzeszów we have started discussing a relationship between null-norms and t-operators. An extended version of this contribution is to be presented at IPMU 2018 in Cádiz. Now, we proceed in this discussion by comparing the notions of divisibility and continuity of monotone operations on bounded posets.

Casasnovas and Mayor [1] introduced the notion of a divisible t-norm and t-conorm. For reader's convinience we repeat that definition. We will assume that $(P, \leq_P, 0, 1)$ is a bounded partially ordered set (poset) with the least element 0 and the greatest element 1.

Definition 1. *A t-norm $T : P^2 \to P$ (t-conorm $S : P^2 \to P$) is divisible if for all $x \in P$ and all $y \leq_P x$ there exists $z \in P$ such that $T(x, z) = y$ ($S(y, z) = x$).*

The notion of divisibility can be generalized for arbitrary commutative monotone (increasing) binary operation $\oplus : P^2 \to P$ in the following way

Definition 2. *A binary commutative operation $\oplus : P^2 \to P$ is divisible if for all $x \in P$, all $y \in [x \oplus 0, x \oplus 1]$ there exists $z \in P$ such that $y = x \oplus z$.*

We see that the notion of divisibility is based only on the partial order $\leq_P$. On every poset P we can consider sequential convergence. This convergence generates a topology on P, so we get a sequential continuity. Of course, also in this case the partial order $\leq_P$ plays a crucial role. However, the two notions – divisibility and continuity – are not identical. Our intention is to compare them. We will be looking for conditions under which divisibility implies continuity, and under which continuity implies divisibility.

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The Topological Spaces Obtained From $C^*_{U,A}$

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In this work, we investigate the topology of $\mathcal{T}_{C^*_{U,A}}$ obtained from the $C^*_{U,A}$ topological closure operator. It is also shown that this topological operator is an algebraic closure operator and $\mathcal{T}_{C^*_{U,A}}$ topological space is Alexandroff Topological space. In addition, it is shown that $\mathcal{T}_{C^*_{U,A}}$ is quasi-Hausdorff space but not Hausdorff space. Moreover, some features of order obtained from topological closure operator $C^*_{U,A}$.

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The Topological Spaces Induced from the Families of the Functions

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A topology given by means of the closure operators of the function families $\{f_i\}_{i \in I}$ is studied. The function families obtained from uninorms is a special case for the defined topology. Especially, in this paper, some properties of the topology is investigated according to the case of uninorms. It is shown that the topology is a quasi-Hausdorff space but not a primal topology. Also, the minimal sets of the topology are determined. The periodic points in the topology is deeply investigated. In this sense, some relationships between the periodic and fixed points are presented.

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An Alexandroff Topology Obtained from Uninorms

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In this paper we obtained an Alexandroff topology and provided some related results, and have given a hint of possible application. Many mathematical systems are lattices and topological spaces at the same time. It is natural to wonder whether the topology in bounded lattices is definable in terms of uninorms. In this paper, the answer to this question is researched, and the presence of such a topology is observed. It is observed that this topology provides the condition of Alexandroff topology. It is shown that this topology can not be metrizable except for the discret metric case. We present an equivalence on the class of uninorms on a bounded lattice based on equality of the topologies induced by uninorms. It is obtained that the set of Alexandroff topologies based on uninorms of the same form has uncountable cardinality.

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Some Properties of the Equivalence Relations Induced by the U-partial Orders

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In this paper, we discuss the equivalences of uninorms on a bounded lattice by means of the equality of the U-partial orders. We determine some relationships between the orders induced by t-norms and their N-dual t-conorms. Also, we present the relations between the equivalence of them. We show that there exists a closely connection between the sets of incomparable elements w.r.t. the orders induced by them. We determine a relationship between the equivalence of two uninorms and the equivalence of their underlying t-norms and t-conorms. We give a necessary and sufficient condition for the equivalence of a uninorm and its conjugate. We investigate the relations between the sets consisting of all incomparable elements w.r.t. the U-partial order and the orders induced by its underlying t-norm and t-conorm.

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An n-ary Generalization of the Concept of Distance

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Generalizations of the concept of distance in which $n \geq 3$ elements are considered have been investigated by several authors (see [1, Chapter 3] and the references therein). The general idea is to provide some functions that measure a degree of dispersion among n points. In this talk, we consider the class of n-distances, which are defined as follows.

Definition 1. *Let X be a nonempty set and $n \geq 2$. A map $d\colon X^n \to [0, +\infty[$ is an n-*distance *on X if it satisfies*

(i) $d(x_1, \ldots, x_n) = 0$ *if and only if* $x_1 = \cdots = x_n$*,*
(ii) $d(x_1, \ldots, x_n) = d(x_{\pi(1)}, \ldots, x_{\pi(n)})$ *for all* $x_1, \ldots, x_n \in X$ *and all* $\pi \in S_n$*,*
(ii) $d(x_1, \ldots, x_n) \leq \sum_{i=1}^{n} d(x_1, \ldots, x_n)_i^z$ *for all* $x_1, \ldots, x_n, z \in X$*,*

where we denote by $d(x_1, \ldots, x_n)_i^z$ the function obtained from $d(x_1, \ldots, x_n)$ by setting its ith variable to z

For an n-distance $d\colon X^n \to [0, +\infty[$, the set of the reals K of $]0, 1]$ for which the condition

$$d(x_1, \ldots, x_n) \ \leq \ K \sum_{i=1}^{n} d(x_1, \ldots, x_n)_i^z \,, \qquad x_1, \ldots, x_n, z \in X,$$

holds has an infimum K^*, called the *best constant associated* with d. The purpose of the talk is to provide natural examples of n-distances based on the Fermat point and geometric constructions, and to provide their best constants. We will also provide examples of n-distances that are not the n-ary part of multidistances as defined in [2].

The results presented in this talk can be found in [3].

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New Approaches for Defining OWA Operators on Partially Ordered Sets

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Our aim in this work is to study two definitions of OWA operators on partially ordered sets, $(P, \preceq)$, closed under convex combination. Notice that, since $\preceq$ is a partial order, there might exist incomparable elements in P and, generally, it is not possible to find a decreasing permutation of a given input vector $(x_1, \ldots, x_n) \in P^n$. This key step has led us to study several approaches for extending aggregation functions based on arrangements of data (such as OWA operators, but also Choquet or Sugeno integrals) to partially ordered sets.

Specifically, we focus on OWA operators defined by means of increasing fuzzy quantfiers $Q : [0,1] \to [0,1]$. We recall that an OWA operator (defined on the usual unit interval $[0,1]$) with respect to Q is a mapping $OWA_Q : [0,1]^n \to [0,1]$ given by

$$OWA_Q(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{(i)}$$

where $x_{(1)} \geq \cdots \geq x_{(n)}$ and $w_i = Q(i/n) - Q((i-1)/n)$ for every $i \in \{1, \ldots, n\}$.

First approach: admissible permutations

The first approach consists in calculating the set of admissible permutations (see [1]) of a given input vector.

Definition 1. *Let $(x_1, \ldots, x_n) \in P^n$. A (decreasing) permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ is said to be an admissible permutation with respect to the partial order $\preceq$ if:*

(i) for every $x_i \prec x_j$, we have that $\sigma^{-1}(j) < \sigma^{-1}(i)$ and
(ii) for each x_i, the set $\{\sigma^{-1}(j) | j \in \{1, \ldots, n\}$ with $x_i = x_j\}$ is an interval in $\mathbb{N}$.

The definition of OWA operators based on admissible permutations is done in two steps. First, given $(x_1, \ldots, x_n) \in P^n$, we calculate the set of admissible permutations, namely $\sigma_1, \ldots, \sigma_p : \{1, \ldots, n\} \to \{1, \ldots, n\}$ (notice that p may vary

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between 1 and $n!$). For each individual admissible permutation σ_j, we calculate

$$OWA_Q^{\sigma_j} = \sum_{i=1}^{n} w_i x_{\sigma_j(i)}$$

with $w_i = Q(i/n) - Q((i-1)/n)$. Second, the OWA operator with respect to every admissible permutation is given by

$$OWA_Q(x_1, \ldots, x_n) = \frac{1}{p} \sum_{j=1}^{p} OWA_Q^{\sigma_j}(x_1, \ldots, x_n).$$

Second approach: sets *L* and *U*

Given $(x_1, \ldots, x_n) \in P^n$, consider the following sets (see [2]) that represent the ordinal structure of the input vector:

$$L(i) = \{r \in \{1, \ldots, n\} | x_r \prec x_i\},$$
$$U(i) = \{r \in \{1, \ldots, n\} | x_i \prec x_r\}.$$

For defining OWA operators based on the sets L and U we first construct a set of intermediate weights given by

$$v_i = \frac{Q((n - |L(i)|)/n) - Q(|U(i)|/n)}{n - |L(i)| - |U(i)|}$$

for every $i \in \{1, \ldots, n\}$. Then, the OWA operator is defined as

$$OWA_Q(x_1, \ldots, x_n) = \frac{\sum_{i=1} v_i x_i}{\sum_{i=1}^{n} v_i}.$$

In this work we will discuss some properties of both approaches and we will analyze similarities and differences among them. We will show some examples to illustrate these properties.

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Joint Aggregation of Rankings and Ordinal labels: A Case Study in Food Science

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The assessment of food quality is one of the most prominent problems in food science. A classical approach to this problem starts by asking several panellists to assign a label on a given ordinal scale to a food sample, subsequently using some aggregation method for combining the labels expressed by all the panellists. Typical examples of such aggregation method are the mode, the median and the mean, the latter being meaningful only in case the given ordinal scale is an interval scale. Note that the latter setting is not the standard in food science (e.g., see the use of hedonic scales [1,3,2,5]), often rendering the use of the mean senseless.

Unfortunately, an appropriate use of a given ordinal scale by a panellist requires exhaustive training, especially when assessing a non-subjective characteristic such as the degree of spoilage of a given food sample. Since this training is expensive and time-consuming, it is quite common to collect datasets with a small number of observations and, thus, it is often difficult to extract meaningful conclusions. Contrarily, in an era in which consumer preferences are daily monitored, some additional information might be available or easily gathered. For instance, invoking a huge number of untrained panellists is way less expensive than training a few panellists. Obviously, there is a trade-off between the cost and the quality of a gathered dataset. However, although an untrained panellist might not be able to appropriately use a given ordinal scale, the panellist might intrinsically be able to rank different food samples. Conceptually, it is easier to state which of two samples is more spoiled than to assess the degree of spoilage of both samples.

For the aforementioned reasons, datasets in which a reduced number of trained panellists assign a label on a given ordinal scale to different food samples and untrained panellists rank these same food samples are often available in the field of food science. Shamefully, the rich nature of these datasets is often disregarded and techniques for aggregating the labels and the rankings are most of the times performed independently. In this presentation, we will discuss how both types of data can be dealt with jointly and illustrate this with a real-life experiment concerning the freshness of different samples of raw Atlantic salmon [4].

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A Computationally Expedient Social-Choice Related Property for Group Decision-Making

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We introduce a computationally expedient social-choice related property for group decision-making, called the Generalized Condorcet Criterion, which can be regarded as a natural extension of the well-known Condorcet criterion [1] and the Extended Condorcet criterion [3]. Unlike its parent properties, the generalized Condorcet criterion is adequate for complete rankings with ties as well as for incomplete rankings. This new property can also provide computational advantages when solving large size problems. Namely, it allows us to simplify the solution process for certain types of instances of the NP-hard Kemeny ranking aggregation problem via a combinatorial branch and bound algorithm. To test the practical implications of this property, we sample complete rankings with and without ties from the Mallows statistical distribution of rank data [2] to generate instances with differing degrees of collective cohesion.

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