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Contents

Aknowledgements

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Algebraic curves over finite fields: rational points and birational invariants

Maria Montanucci, Technical University of Denmark

Algebraic curves over a finite field $\mathbb{F}_q$ have been a source of great fascination, ever since the seminal work of Hasse and Weil in the 1930s and 1940s. Many fruitful ideas have arisen out of this area, where number theory and algebraic geometry meet, and many applications of the theory of algebraic curves have been discovered during the last decades. A very important example of such application was provided in 1977-1982 by Goppa, who found a way to use algebraic curves in coding theory. The key point of Goppa's construction is that the code parameters are essentially expressed in terms of the features of the curve, such as the number N_q of $\mathbb{F}_q$-rational points and the genus g. In this light, Goppa codes with good parameters are constructed from curves with large N_q with respect to their genus g.

Given a smooth projective, algebraic curve of genus g over $\mathbb{F}_q$, an upper bound for N_q is a corollary to the celebrated Hasse-Weil Theorem,

$$N_q \leq q + 1 + 2g\sqrt{q}.$$

Curves attaining this bound are called $\mathbb{F}_q$-maximal.

The Hermitian curve is a key example of an $\mathbb{F}_q$-maximal curve, as it is the unique curve, up to isomorphism, attaining the maximum possible genus of an $\mathbb{F}_q$-maximal curve. It is a result commonly attributed to Serre that any curve which is $\mathbb{F}_q$-covered by an $\mathbb{F}_q$-maximal curve is still $\mathbb{F}_q$-maximal. In particular, quotient curves of $\mathbb{F}_q$-maximal curves are $\mathbb{F}_q$-maximal. Many examples of $\mathbb{F}_q$-maximal curves have been constructed as quotient curves of the Hermitian curve by choosing a subgroup of its very large automorphism group.

It is a challenging problem to construct maximal curves that cannot be obtained in this way, as well as to construct maximal curves with many automorphisms (in order to use the machinery described above). A natural question arises also: given two maximal curves over the same finite field, how can one decide whether they are isomorphic or not? A way to try to give an answer to this question is to look at the birational invariants of the two curves, that is, their properties that are invariant under isomorphism.

In this talk, we will describe our main contributions to the theory of maximal curves over finite fields and their applications to coding theory. In relation with the question described before, during the talk, the behaviour of the birational invariant of maximal curves will also be discussed.

Nonautonomous dynamical systems, critical transitions and nonautonomous bifurcations

Iacopo P. Longo, Imperial College London

In this talk, I introduce the theory of nonautonomous dynamical systems and review some very recent results that allowed to connect nonautonomous bifurcation theory to critical transitions.

Imagine having a ball resting on the palm of your open hand. If you move slowly, the ball will keep up and maintain its relative position. However, if you quickly sweep your arm, you'll see the ball irremediably rolling out of your palm and falling on the floor. Yet, the palm of your hand hasn't changed through the process; As a topological object, it doesn't seem to have lost its properties of stability. What is the cause of the collapse in the state of the ball then? This is a simple example of what we call a critical transition.

Traditional mathematical theories successfully describe transitions under the assumption that environmental conditions change slowly ("adiabatically") with respect to the dynamics of the system. This is the case of autonomous bifurcation theory for low-dimensional autonomous dynamical systems and statistical mechanics for very large homogeneous systems. However, the assumption of an adiabatically changing environment is often not satisfied in practice since parameters vary at a timescale comparable to the internal dynamics. Consequently, new scenarios can be observed that cannot be explained with the autonomous theory, such as critical transitions. I employ nonautonomous dynamical systems theory to bridge this gap.

Ultrametric models for dimensionality reduction in single and heterogeneous populations

Giorgia Zaccaria, University of Milano-Bicocca

Multidimensional phenomena such as well-being, climate change, sustainable development, and poverty are often defined by nested latent concepts, which can be represented through a tree-shaped structure, assuming hierarchical relationships among variables. The class of ultrametric models introduced here offers methodologies suitable for studying these relationships using a simultaneous and exploratory approach. The pivotal feature of these models is parsimony, as they identify a partition of the variable space into disjoint groups and explore their hierarchical relationships. Starting with the definition of an *ultrametric matrix* (Dellacherie et al., 2014), we present the Ultrametric Correlation Model (Cavicchia et al., 2020), which reconstructs an observed data correlation matrix using an ultrametric one. This model is then extended to Ultrametric Factor Analysis, which identifies the hierarchy of latent concepts and quantifies them through factors.

While the nonnegativity assumption of ultrametric matrices is realistic in many applications, such as psychometric studies, it proves too restrictive in others. To address this issue, we introduce an extended ultrametric matrix (Cavicchia et al., 2022), which accommodates both positive and negative relationships among variable groups by aggregating them from the most concordant to the most discordant. This structure has been used to model covariance matrices in Gaussian mixture models, providing a different characterization of a multidimensional phenomenon in heterogeneous populations. By constraining structures within and across components, we developed 13 Parsimonious Ultrametric Gaussian Mixture Models (Cavicchia et al., 2024), which further reduce the number of parameters needed to model relationships among variables. Finally, the Ultrametric Gaussian Mixtures are extended to handle missing data (Greselin and Zaccaria, 2024).

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Function spaces between smooth and analytic classes

Javier Jiménez-Garrido, University of Cantabria

In the early 20th century, it was observed that certain problems in differential equations could be more naturally and conveniently studied by introducing intermediate spaces between analytic and smooth function spaces. These intermediate spaces were defined by limiting the growth rate of their elements' derivatives through a sequence of positive real numbers, known as a weight sequence. Similarly, the formal power series solutions for algebraic differential equations also exhibited controlled growth in their coefficients. These insights motivated the definition of Carleman ultradifferentiable (in the real case) and ultraholomorphic (in the complex domain) classes. In particular, Carleman-Roumieu ultraholomorphic classes in sectors are closely related to the concept of asymptotic expansion.

In this talk, we will discuss several aspects of the study of these classes. Specifically, we will focus on results concerning the nontriviality, injectivity, and surjectivity of the corresponding Borel-Peano map, as well as (mixed) extension results, and stability properties. If time permits, we will explore how similar classes can be presented using weight functions or weight matrices, and we will highlight the new ideas that arise from this perspective.

Parallel sessions

Inflection points of algebraic surfaces, a modern approach to the work of George Salmon

Jorge Rodríguez Pérez, University of Valladolid

Given an algebraic surface in complex three-dimensional projective space and a line passing through a point on this variety, the order of contact of the line with the surface can be defined as detailed in the article [2]. We call inflection points the points on the surface for which there exists a line with an order of contact greater than 3. In his book "A treatise on the analytic geometry of three dimensions" [3], George Salmon states that for an algebraic surface of degree d greater than or equal to 3, there exists a homogeneous polynomial of degree $11d - 24$ that determines its inflection points. Furthermore, he provides a method to calculate it which is due to Clebsch [1]. However, the use of techniques that have fallen into disuse and the outdated notation used in their publications, makes Clebsch's calculation a result that is not fully understood today.

The main purpose of this work is to provide a modern theoretical framework for Clebsch's calculation, following his steps while addressing any potential oversights due to a lack of rigour. A key technique in this process is the symbolic method, which involves using symbols to operate on homogeneous polynomials, and is formalized using multilinear algebra in a similar way as it is done by Dolgachev [4]. Another important concept in Clebsch's calculation is the use of polars, which are surfaces derived from the original that allow to derive its local properties from the intersection between them. With this two tools, the calculation performed and a closed expression for Salmon's polynomial is provided.

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Tropical ideals

Gonzalo Rodríguez Pajares, Universidad de Valladolid

Tropical ideals form a class of ideals defined in the semiring of tropical polynomials, as introduced in [1]. These ideals include the tropicalizations of classical ideals and show a close connection to valuated matroids. We will investigate that relation and show the most important properties and results concerning tropical ideals. In particular, we will show that, similar to the classical framework, it is possible to define a Hilbert function for these ideals. Every tropical ideal has an associated variety, which we will demonstrate is a finite polyhedral complex. We will also prove that they satisfy the ascending chain condition, although they are not finitely generated, and we show that they satisfy a tropical version of Nullstellensatz.

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Stratification of the moduli space of plane branches with a single characteristic exponent

María de Leyva Elola-Olaso, Universitat Politècnica de Catalunya

We study the moduli space of plane branches with a single characteristic exponent through a stratification using the semimodule of values of the Jacobian ideal of the branch. We provide an algorithmic procedure to describe the strata and their dimensions. This stratification refines a previously known one based on the Zariski invariant studied by Peraire in 1998.

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An introduction to the space of arcs

Miguel Valderrama de las Heras, Universidad de Valladolid

The concept of the space of arcs of a variety was introduced by John Nash in the 1960s, shortly after H. Hironaka's proof of the resolution of singularities in characteristic 0. Nash's goal was to recover properties of the resolution of singularities of an algebraic variety over a field of characteristic 0 by studying certain invariants in its space of arcs. His work in this field was made known by H. Hironaka in the 1970s and later by M. Lejeune-Jalabert.

In this work, we take some initial steps in the study of this field, where many questions remain unanswered to this day, even in the simplest case: the cusp. The objective is to understand the concept of the space of arcs of a variety and its construction, and to study some of its properties, using the cusp as an example. Finally, we will raise some of these open questions and attempt to address them in the case of the cusp, ultimately obtaining the answer to one of them, finding a relation between the space of arcs of this variety and its normalization.

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Computer Science Through the Frequency Spectrum

Isabel María Moreno Cuadrado, Universidad Complutense de Madrid

In an era where explainability in Deep Learning (DL) is becoming increasingly significant, understanding how these mathematical frameworks can demystify the decision-making processes of DL models is vital. This talk will demonstrate how the mathematical part supports and motivates new advances in a line of research within DL, thus connecting both disciplines and illustrating the fruitful results that can arise when they are combined.

In this talk, structured in two distinct parts—one of mathematical nature and the other of computational nature—some issues related to the field of DL, specifically Convolutional Neural Networks (CNN), are explored, establishing an interconnection between both areas.

In the mathematical part of this work, corresponding to the first section, Fourier analysis is studied in depth. The insights gained from this analysis are then applied to studying image filtering using its frequency representation, leading to a better understanding of these processes.

Additionally, a well-known alternative method for performing the convolution operation in CNN between a kernel and an image is examined [1], utilizing the Convolution Theorem and the properties of the Discrete Fourier Transform (DFT). This method employs the Fast Fourier Transform (FFT) [1] to convert the matrix products between both operands into pointwise products. Subsequently, the efficiency of this algorithm is analyzed, requiring a detailed study of the FFT algorithm's performance.

This approach allows training a CNN using fewer operations and, therefore, represents a promising advancement in the line of research aimed at accelerating CNN training. This advancement is especially relevant given the increase in the number of feature maps in modern CNN, which also tend to work with large-scale data. This methodology aligns with the growing awareness of "Green AI," which seeks to develop sustainable AI technologies and techniques, promoting a more efficient and responsible approach.

This work highlights the connection between mathematical theory and computational applications, bridging the gap between Fourier analysis and its practical uses in various fields, including signal processing and, more recently, DL.

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Computational study of neural networks using topological invariants

Jon Ander Alonso, Universidad de La Rioja

Eduardo Sáenz de Cabezón, Universidad de La Rioja

Topological data analysis (TDA) is a branch of data science that uses topological tools to analyze datasets, through techniques like persistent homology. This methodology has already achieved success in various fields of science and engineering. In recent years, it has begun to be applied to machine learning (ML), particularly to the analysis of models. It may provide insights into open problems such as generalization capacity or explainability. The relationship between topology and these properties is an active area of study, as described in [1].

We present an experimental study of the topological evolution of a simple multilayer perceptron during its training, as it gradually reaches overfitting. At each epoch, we apply persistent homology to the activations of each layer while processing training and test data, capturing the topological statistics described in [2]. The results suggest relationships between the model's learning curve and these statistics, as well as differences in topological complexity between successive layers.

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Neural Network-Enhanced Finite Volume Schemes for Hyperbolic System of Conservation Laws in Multiple Dimensions

León Miguel Ávila León, Universidad de Málaga

Manuel Jesús Castro Díaz, Universidad de Málaga

The rapid evolution of Physics Informed Neural Networks (PINNs) [1] and Variational PINNs (VPINNs) [2] signifies a transformative shift in the field of computational mathematics, particularly in solving complex non-linear partial differential equations (PDEs). These methodologies efficiently encode physical laws into the architecture of neural networks, allowing the numerical approximation of PDEs.

In this context, we propose to combine PINNs and the traditional finite volume methods (FVM), which are widely used for numerical approximations of hyperbolic conservation laws. First, we consider the integral form of the hyperbolic conservation law and a suitable partition of our computational mesh into cells. Next, we will use a neural networks, with as many input neurons as cells, as a reconstruction operator in the FVM framework. The role of the neural network is also involved in the calculation of the solution averages in the cells for input into a loss function related to the finite volume schemes.

The advantage of using neural networks is to be able to consider implicit methods without increasing the complexity, which allows us to increase stability and to be able to use larger time steps. The neural network performs the crucial task of reconstructing intercell fluxes. The way the method is assembled will allow us to construct in the future well-balanced methods in this framework [3], or to make use of entropy-conservative numerical fluxes.

Moreover, the efficiency of the proposed method will be demonstrated through its application to a variety of hyperbolic conservation laws, including transport equations, Burgers' equations and Shallow Water equations, both in 1D and 2D.

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Opinion formation under the presence of ignorance in social groups

Mencía Reborido, Universitat Politècnica de Catalunya

Opinion dynamics is a sub-field of social dynamics that studies the exchange of opinions between individuals under certain social conditions and interconnections. By means of dynamical systems, we can predict how opinions evolve over time, and whether a consensus would be reached in a particular social network. As opposed to the usual settings assumed in the typical physical models, we introduce the key figure of *the uninformed* individual, whose affinity for any opinion present in our spectrum is null. By means of this modified system, we study how opinions evolve under several circumstances.

Axioms for the Lefschetz number on sheaves

Alejandro Majadas Moure, Universidad de Santiago de Compostela

David Mosquera Lois, Universidad de Vigo

The Lefschetz number is a very important invariant in algebraic topology. It is also the generalization of the Euler characteristic for maps different from zero. A powerful tool of the Euler characteristic is the relationship that it establishes between the topological context and the data context (sheaves) ([2]). In this work, we generalize this relationship to the Lefschetz number and we obtain some axioms that characterize this invariant in the language of sheaves and reduce the usual axioms for the Lefschetz number ([1]).

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Sheaf cohomology and applications

Alfonso Márquez Martínez, Universidad de Sevilla

This talk seeks to be an introduction to sheaf cohomology, defining the concept through derived functors in order to show some of the main results in the area, such as a vanishing theorem by Grothendieck or Serre´s Duality for projective schemes, which is our main result. We will also introduce Čech cohomology, which simplifies some computations, and show remarkable applications of this theory.

Cohomology Lusternik-Schnirelmann Category

Ángel Méndez, Universidade de Santiago de Compostela

Enrique Macías, Universidade de Santiago de Compostela

David Mosquera, Universidade de Santiago de Compostela

The Lusternik-Schnirelmann category of a topological space is a well-known homotopy invariant, yet difficult to calculate. It originally appeared in variational calculus as a lower bound for the number of critical points of a function defined on a manifold. Recently, it has gained popularity due to its connection with some problems related to robotics. As a result, different versions and generalizations have emerged in finite, combinatorial, geometric, simplicial, and small category contexts. The aim of the talk is to present a lower bound: the cohomology category. We will also discuss the simplicial category and its corresponding cohomology version, which can be calculated using computational methods for any triangulable space.

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The Abstract De Rham Theorem for Sheaf Cohomology. Some Examples.

Gabriel Abánades, Universidad Complutense de Madrid, Instituto de Ciencias Matemáticas

The goal of this talk is to introduce a result in sheaf cohomology sometimes referred to as *The abstract de Rham Theorem*. This result states that any acyclic resolution of a sheaf computes its cohomology. To contextualize this result we will introduce the notion of flasque, soft and fine sheaves, these being the most important examples of acyclic sheaves and will be used during the presentation.

This theorem is of great importance as it provides many isomorphisms between cohomology groups otherwise defined using very different approaches. As an example we will show the fact that the de Rham cohomology of a smooth manifold agrees with its real singular cohomology (as one can see in [1]), which is known as the classical de Rham Theorem. The theorem also implies the fact that for a manifold endowed with an integrable almost complex structure, the Dolbeault cohomology computes the cohomology of the sheaf of p forms (as one can see in [2]). That is

$$\mathrm{H}^{p,q}(X) \cong \mathrm{H}^{q}(X, \Omega^{p}).$$

If time allows, additionally we will give a short proof of the theorem using the degeneray of a spectral sequence that arises when studying double complexes, as well as an alternate longer proof which does not use spectral sequences.

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On QMDS codes

Rubén Rodríguez Ballesteros, Universidad de Valladolid

Umberto Martínez Peñas, Universidad de Valladolid

In this work, we study linear codes endowed with the block Hamming distance, or equivalently, codes with the classical Hamming distance that are linear over a subfield. This includes additive codes. We study MDS codes in this setting and define quasi-MDS (QMDS) codes and dually QMDS codes, which attain a more relaxed variant of the classical Singleton bound. We then provide MacWilliams equations and the weight distribution of dually QMDS codes, which only depends on their parameters and not on the codes. We conclude with an lower bound on the field size of dually QMDS codes.

Optimising Algebraic Algorithms using Artificial Intelligence

Tereso del Río Almajano, Coventry University

Artificial intelligence models are stochastic by nature and thus may seem useless for obtaining exact results like those provided by algebraic algorithms.

However, algebraic algorithms make multiple decisions that do not affect the validity of the result, such as monomial order.

To make these decisions, often heuristics created by experts are used. However, artificial intelligence models have been able to outperform experts in almost every imaginable field, and this is no different in the topic at hand [2]. In this talk, we will explore how artificial intelligence models can be trained to make these decisions.

This talk will serve as a summary of my thesis [3], during which I have worked on designing ways to choose a good variable order for the Cylindrical Algebraic Decomposition algorithm proposed by Collins in 1975 [1]. At the beginning of my thesis, I proposed a heuristic for choosing the variable order, then I worked on training artificial intelligence models to make these decisions, and finally, using model interpretation techniques, I extracted simple heuristics from complex artificial intelligence models that became the state-of-the-art.

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Complexes of groups

Marina Salamero, Universidad de Sevilla

Bass and Serre's well known theory of groups acting on trees [3] tells us how one can retrieve a presentation of the group from the orbit space and the stabilizers of the vertices and the edges. But a tree is nothing but a 1-dimensional simply connected complex, which leads us to the following question: is there a way to generalize this theory to higher dimensions?

In this talk, following the book of Bridson and Haefliger [1], we will see that indeed it is possible to apply the same techniques to study groups acting without inversions on cell complexes with a strict fundamental domain. For the sake of concision, we will restrict our attention to simplicial complexes, for which the cells can be easily labeled by means of a poset that will encode the inclusions of the corresponding stabilizers, although the theory can be developed in the wider context of so-called stratified spaces. We will as well show a recent application of these techniques to a problem on parabolic subgroups of Artin groups [2].

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Residual Finiteness Growth

Joren Matthys, KU Leuven

A group G is residually finite if for every non-trivial element $g \in G$ we can find a homomorphism $\varphi : G \to Q$ to a finite group Q such that $\varphi(g) \neq 1$. Equivalently, the profinite topology on G is Hausdorff. This property is shared by many finitely generated groups found in other fields, e.g. free groups, linear groups, nilpotent groups, fundamental groups of compact 3-manifolds ...

In 2010, Bou-Rabee quantified this property for finitely generated, residually finite groups: he introduced the (normal) residual finiteness growth [1]. Given $g \in G$, it estimates the minimal order, $D(g)$, of Q in the definition above in terms of $\|g\|_S$. Here, $\|\cdot\|_S$ denotes the word metric on G corresponding to some finite generating set S. More precisely, the residual finiteness growth is the non-decreasing function $\mathrm{RF}_G : \mathbb{N} \to \mathbb{N} : n \mapsto \max\{D(g) \mid 0 < \|g\|_S \leq n\}$ with $D(g) = \min\{|Q| \mid 1 \neq \varphi(g) \text{ for } \varphi : G \to Q\}$. The asymptotic behaviour of this function is a group invariant, e.g. for $\mathbb{Z}$ we have logarithmic growth ($\mathrm{RF} \approx \log$) due to the Prime Number Theorem.

As listed in the survey [2], bounds on the function RF have been established for several classes. However, exact results are mostly lacking, most notably there is no exact result yet for the free groups, neither for some other traditional classes such as nilpotent groups.

Recently, however, we managed to show that RF_G grows like $\log^k$ for virtually abelian groups [3]. Here, k can be explicitly determined via the representation theory of finite groups. Its proof has provided inspiration for further progress in the field of nilpotent and polycyclic groups. We highlight some of the general ideas/notions/properties that affect the residual finiteness growth of groups.

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Projective tensor products where every element is norm-attaining

Juan Guerrero-Viu, University of Zaragoza

Luis C. García-Lirola, University of Zaragoza

Abraham Rueda Zoca, University of Granada

Given X, Y two Banach spaces, we can consider the projective tensor product $X\widehat{\otimes}_\pi Y$ which is used to linearize bilinear functions. In this talk we analyse when every element of $X\widehat{\otimes}_\pi Y$ attains its projective norm. We prove that this is the case if X is the dual of a subspace of a predual of an $\ell_1(I)$ space and Y is 1-complemented in its bidual under approximation properties assumptions. This result allows us to provide some new examples where X is a Lipschitz-free space.

Algebraic dimension of completely metrizable topological vector spaces

Mario P. Maletzki, Universitat Jaume I

The problem of determining if for a given vector space V there is a completely metrizable topology compatible with the algebraic structure is approached, showing in particular that the problem is determined by the cofinality of $dim(V)$ if and only if the *Singular Cardinal Hypothesis* holds.

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On L_p Brunn-Minkowski type inequalities for a general class of functionals

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Jesús Yepes Nicolás, Universidad de Murcia

The L_p version (for $p \geq 1$) of the well-known Brunn-Minkowski inequality, originally proven by Firey in the 60's for convex bodies containing the origin, and recently extended to arbitrary non-empty compact sets by Lutwak, Yang and Zhang, asserts that

$$\mathrm{vol}\big((1-\lambda)\cdot K +_p \lambda \cdot L\big)^{p/n} \geq (1-\lambda)\mathrm{vol}(K)^{p/n} + \lambda \mathrm{vol}(L)^{p/n},$$

for any non-empty compact sets $K, L \subset \mathbb{R}^n$ and any $\lambda \in (0,1)$. Here $\mathrm{vol}(\cdot)$ denotes the Lebesgue measure on $\mathbb{R}^n$, and $+_p$ is the p-addition of the sets there involved (operation which comprises the Minkowski sum when $p = 1$).

In this talk we will discuss different Brunn-Minkowski type inequalities for a general class of functionals, defined on the family of convex bodies, when dealing with the p-sum of the sets involved. As a particular case of our approach, we will derive new L_p Brunn-Minkowski type inequalities for both the standard Gaussian measure and the so-called Wills functional (i.e., the sum of the intrinsic volumes), tools of high interest in Convex Geometric Analysis, as shown by the amount of different recent research works on them.

New optimal estimates in Hardy-type inequalities

Achraf Ben Said Abid, Universidad Complutense de Madrid

In this presentation we intend to illustrate in a brief way the study of optimal constants in some inequalities in L^p norm of operators involving the Hardy operator defined as

$$Hf(x) = \frac{1}{x}\int_0^x f(t)dt.$$

In 1925, Hardy [3] proves that H is bounded from L^p to L^p, if $1 < p \leq \infty$, with norm exactly equal to p'. This result has given rise to a new field of study of great interest in Mathematical Analysis and Operator Theory. The aim of this talk is to go into this line of research by exposing the results that have been obtained recently [2, 4, 5, 6, 7], the open problems in this field, such as the problem formulated in 1996 by G. Bennett [1] and, finally, to present some of our contributions giving solutions to several questions within this subject.

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Convex bodies associated to log-concave functions

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The relationship between the world of convex bodies and the world of log-concave functions is fairly close. A case in point are the so-called Ball bodies, that are sets defined via some integrable function, and whose study and use is the main goal of the talk. We will prove that those sets are, in fact, convex when the defining function is log-concave. Moreover, certain inclusion relations between those bodies will be studied, we will see some results linking Ball bodies with super-level sets, and as an application we will see how to reduce the hyperplane conjecture, a problem that has remained unsolved for the last 30 years. What we will prove is that, should that conjecture be true for centrally symmetric convex bodies, it will automatically be true for any convex body.

A literature review on how to develop computational thinking from a geometrical perspective using unplugged activities with games

Gregorio Arjona-Aranda, University of Málaga

Given the recent inclusion of computational thinking in the mathematical curriculum from the Spanish education law, teachers all across tbe country are left with a challenge to which they do not feel prepared [2]. It is necessary to build guiding tools that help them with the development of computational thinking skills in students of any level.

Computational thinking can be developed through mathematics because it implies problem resolution, system designs, and a comprehension on how people behave through ideas taken from information technology [3]. It usually is linked with algebra, since it showcases pattern recognition and generalization, two key concepts from this branch of mathematics [1].

In this talk, we develop a literary review with the intent to broaden computational thinking to other branches like, in this instance, geometry. We will also check the validity to use game-based learning (GBL) as a methodology to spread computational thinking. We will link some widely accepted computational thinking skills with elements from GBL and geometry, using a game that can be easily accesible and easily understandable to play with groups of people.

We will also present a game that we can use to develop an intervention with a set of activities, taking previous experiences that used GBL to teach geometry and/or develop computational thinking skills into consideration. With this game, we hope that students develop their sense of abstraction by creating complex figures using simple forms.

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Navigating the turbulent waters of math anxiety: A meta-analysis of interventions for pre-service teachers

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Despite the crucial role that mathematical competence plays in both daily and professional aspects of our lives, set within a technological and globalized society [2], reports like [3] reveal that students in our educational system demonstrate inadequate proficiency in this area a problem that has its roots, among other factors, in their high levels of math anxiety, which have been shown to correlate negatively with performance [1, 3, 4]. Since this emotion can be linked to teaching practices and the math anxiety experienced by teachers themselves [1, 4], it makes sense to address this problem at its source. Therefore, with the aim of laying the groundwork for a future intervention, we conducted a meta-analysis to evaluate interventions aimed at reducing math anxiety among pre-service teachers, with the goal of identifying the characteristics of the most successful programmes. The results indicate that the effectiveness of the interventions increases with their duration, with the largest effect size being observed in those that utilized active methodologies.

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Tasks of defining polygons for prospective teachers

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Matemathics´s teacher knowledge study the knowledge of a teacher to teach mathematics in class. In the University of Huelva, the knowledge of mathematics teachers is studied with the *Mathematics Teacher´s Specialized Knowledge* model (MTSK). This model includes both mathematics knowledge, pedagogical content knowledge and beliefs about knowledge (Carrillo-Yañez et al., 2018). MTSK model uses as an analytic tool and as well as for designing formative tasks. In the research project "Specialized knowledge in formation of mathematics teachers: Tasks and knowledge of formator (MTSK-T&MTEK)" it is been used like a tool for designing formative tasks which develop specialized knowledge of future teachers. As example of a tasks designed and implemented is a task which consist in writing a defining of polygon and reflecting about the practice of defining. Cartoons and questions guide to future teachers to reflect on definitions. The specialized knowledge associated with practice of defining is related to knowledge of knowledge of practice in mathematics. (KPM, Climent et al. 2024). A good definition of polygons according to Winicki-Landman and Leikin (2000) must have some principles: defining is give a name; to defining a new concept only concepts defined previously can be used; a definition establishes some necessaries and sufficient conditions to concept; set of conditions must be minimal; a definition is arbitrary. Future teachers learn finally these principles with this task.

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Mode-based estimation of the center of symmetry

Javier Fernández Serrano, Universidad Autónoma de Madrid

José E. Chacón, Universidad de Extremadura

In the mean-median-mode triad of univariate centrality measures, the mode has been overlooked for estimating the center of symmetry in continuous and unimodal settings. This paper expands on the connection between kernel mode estimators and M-estimators for location, bridging the gap between the nonparametrics and robust statistics communities. The variance of modal estimators is studied in terms of a bandwidth parameter, establishing conditions for an optimal solution that outperforms the household sample mean. A purely nonparametric approach is adopted, modeling heavy-tailedness through regular variation. The results lead to an estimator proposal that includes a novel one-parameter family of kernels with compact support, offering extra robustness and efficiency. The effectiveness and versatility of the new method are demonstrated in a real-world case study and a thorough simulation study, comparing favorably to traditional and more competitive alternatives. Several myths about the mode are clarified along the way, reopening the quest for flexible and efficient nonparametric estimators.

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Step-Stress Tests for Highly Reliable Devices

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Many modern products are extremely reliable with long lifetimes to failure. Reliability tests under normal operating conditions would require very long experimental times (sometimes infeasible) for accurate inference. Instead, accelerated life tests (ALTs) are used to reduce the lifetime of devices by increasing one or more stress factors, which induce failure. In particular, in step-stress tests the stress applied to a product (e.g., temperature, voltage, pressure) is increased in steps or stages, typically after predefined time intervals or until failure is observed at each stage. Each step exposes the product to higher levels of stress than the previous one. After suitable inference, results can be extrapolated to working conditions. The collected failure data is used to estimate reliability metrics such as mean time to failure or failure rate, helping manufacturers make better decisions about product design and warranties. However, some observed failure times may be censored, while other failure times are continuously recorded. Then, a model relating both types of observations is required.

Classical estimation methods relying on the likelihood function of the lifetime distribution can be significantly influenced by data contamination. As an alternative, robust estimators based on distance measures have been recently developed. In this talk, we develop robust inferential methods based on the density power divergence for inferring the lifetime distribution of highly reliable products.

Imperfect maintenance and parameter estimation in a Wiener process

Lucía Bautista, Universidad de Extremadura

Inma T. Castro, Universidad de Extremadura

Industrial systems are becoming increasingly complex; therefore, they are likely to suffer from multiple degradation. Practical examples can be found in the fatigue cracks of two terminals of an electronic device or in heavy machine tools, among others. These systems are subject to maintenance actions to reduce their level of deterioration. It is essential to analyse systems with more than one degradation path and their dependences to seek the maintenance policy that best suit them. In this work, the underlying degradation process is modelled as a Wiener process, while the effects of maintenance are assumed to be imperfect, described by an Arithmetic Reduction of Degradation (ARD) model. ARD regularly reduces system degradation by an amount proportional to the degradation state just prior to maintenance actions. The Pearson correlation coefficient between the processes is computed and some properties of the monotonicity of this coefficient are studied, as well as the statistical inference in presence of imperfect maintenance actions.

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Alexandrov-Fenchel inequality

Beatriz Marín Gimeno, Universidad de Murcia

If we consider the volume, denoted by $\mathrm{vol}(\cdot)$, of a positive linear combination of convex bodies, which are non-empty convex compact sets , we can expand it as a polynomial whose coefficients define another functional called mixed volumes, denoted by $V(\cdot, \ldots, \cdot)$.

An important result for mixed volumes is the second Minkowski inequality which is given by

$$V(K, \ldots, K, L)^2 \geq \mathrm{vol}(K)V(K, \ldots, K, L, L), \tag{1}$$

where K and L are convex bodies. Besides, we can notice that K is repeated $n-1$ times in the left-hand side.

In this talk we are going to see some properties of mixed volumes, a generalization of (1) which is called Alexandrov-Fenchel inequality that considers n arbitrary convex bodies (not necessarily repeated), the relationship between (1) and Brunn-Minkowski inequality and some particular equality cases since it is not characterized.

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Dynamical behavior of the Takagi function

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Zoltán Buczolich, Eötvös Loránd University

The Takagi function is a classical example of a continuous nowhere differentiable function. It is defined as

$$T(x) = \sum_{n=0}^{\infty} \frac{\phi(2^n x)}{2^n}, \quad x \in [0,1]$$

where $\phi(x)$ denotes the distance from the point x to the nearest integer.

In this talk, we study the discrete dynamical system generated by the Takagi function, namely the one-dimensional dynamical system given by

$$x_{n+1} = T(x_n), \quad x_0 \in [0,1].$$

Firstly, we describe the behavior of the orbit $(T^n(x_0))_n$ for any seed $x_0 \in [0,1]$. The notation T^n represents the composition of T with itself n times.

The value of the Takagi function at a point can be numerically calculated by a computer. However, this value will always be an approximation of the real value because the computer can only sum a finite number of terms in the series defining the Takagi function. For this reason, it is natural to ask whether numerical solutions can be trusted to represent the real orbits of the discrete dynamical system presented above. This leads us to study the shadowing property for the Takagi function.

A characterization of extremal punctured hyperbolic surfaces

Claudia Muñoz, Universidad Autónoma de Madrid

A hyperbolic surface is just a topological surface whose universal covering is the upper-half plane $\mathbb{H}$, so that it can be realized as a quotient of $\mathbb{H}$ by a group of isometries of the upper-half plane whose action on $\mathbb{H}$ is free and properly discontinuous. Hyperbolic surfaces have a natural metric inherited from the hyperbolic metric on $\mathbb{H}$, so we can embed metric discs on them and study some properties of these discs.

Particularly, it would be natural wondering about the maximum radius of a disc embedded in a certain hyperbolic surface. The starting point of the study of bounds for the radius of disc packings on hyperbolic surfaces was in 1996, when Christophe Bavard showed a sharp upper bound r_g for the radius of a metric disc embedded on a compact Riemann surface of genus g, for each $g \geq 2$.

In this talk, after establishing the preliminaries needed, I will introduce the Bavard's bound and comment on some of its generalizations: firstly, an upper bound for the radius of a packing of k equal radius discs on any compact hyperbolic surface of Euler characteristic χ, and then, an upper bound admissible for non-compact hyperbolic surfaces of finite area too, case in which the bound is proven using a really deep theory introduced by Jason DeBlois.

Moreover, I will comment on some characterizations of the k-extremal compact hyperbolic surfaces, that is to say, those surfaces which contain a k-packing of discs whose radius attain the previous upper bound. Specifically, we will talk about a characterization of the groups of isometries associated with these k-extremal surfaces and their fundamental domains. Finally, I will address similar questions for 1-extremal, non-compact hyperbolic surfaces through some results that I have worked on with Ernesto Girondo, the director of my Master Dissertation.

An abstract approach to differential geometry: the compatibility equations

José S. Santiago, University of Jaén

Traditionally, the geometry of surfaces in $\mathbb{R}^3$ is introduced by analyzing their local behavior within this ambient space. These definitions are typically tied to other objects or elements presented in the surrounding geometry.

In this talk, we will study the compatibility equations, which offer a way to encode the geometry of a surface in $\mathbb{R}^3$ solely through data defined on the surface itself. This approach allows us to determine the shape and geometry of the surface using only "intrinsic" information. We will explore how many classical theorems in surface geometry can be straightforwardly proved from this perspective and examine how this viewpoint connects to other areas, such as complex analysis.

Affective-cognitive profiles in students with dyscalculia: a comparative analysis

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Marbán, J. M., Universidad de Valladolid

Maroto, A. I., Universidad de Valladolid

Dyscalculia is a learning disability that persistently affects the acquisition of mathematical skills and it presents itself in a very heterogeneous way among students. It is crucial that educational intervention programmes adequately address the heterogeneity of profiles from personalised designs [2]. The aim of our study was to identify the profiles that allow a deeper understanding of the different ways in which dyscalculia manifests itself in Spanish primary school students from both a cognitive and affective point of view. Data collection was carried out on a sample of 22 cases diagnosed with dyscalculia with the application of a checklist designed and validated ad hoc by the authors for the detection of dyscalculia risk symptoms [1] and the Spanish adaptation of the well-known affective scale Math and Me. Four affective-cognitive profiles have been obtained and the results confirm the great variability. In this talk we will focus on the comparison of our results with those obtained by other researchers with the same research objective. Although previous studies have been carried out on samples of different ages and characteristics and have used different instruments for data collection, it has been decided to seek whether the characteristics of the profiles obtained in this research have any similarity with those of these investigations. The criterion followed for the pairing of subtypes or profiles was the degree of similarity in relation to the type of difficulties most commonly exhibited by students belonging to that subtype. The results of this comparative analysis have shown that two of our profiles are the ones most frequently associated with the characteristics of the subtypes obtained in previous studies, as the cases that form them manifest a wide variety of mathematical difficulties, while the other two present a smaller variety. In addition, our study includes the identification of the affective profile of students with dyscalculia, in contrast to previous studies.

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Limiting Beliefs in Mathematics Education among Primary School Students: A Creative Systematic Review and construction of the concept

Mercedes Carpintero Gómez, University of Valladolid

The research addresses the issue of limiting beliefs among primary education students. It has been observed that, despite having sufficient competencies, many students experience problems and recurring difficulties in solving mathematical problems, possibly due to insecurities or fear of failure based on previous beliefs.

The study focuses on beliefs, a less explored aspect within the affective domain in mathematics, analyzing their fluctuating nature and their impact on learning and classroom participation. Questions are raised regarding the definition, characteristics, and malleability of these beliefs, as well as the influence of teaching practices and the learning context. Through a systematic and creative review of literature, including a bibliometric analysis and critical examination of 40 selected documents, the study aims to provide an updated operational definition of limiting beliefs in mathematics and synthesize the different meanings of the concept. The main objective is to acquire a better understand how these beliefs affect mathematical learning and explore possible strategies for their modification or prevention, in order to improve students' affective relationship with mathematics.

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Dealing with non-principal locus in automated reasoning in geometry

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Tomás Recio, Universidad Antonio de Nebrija

M. Pilar Vélez, Universidad Antonio de Nebrija

The integration of the computer algebra system Giac into GeoGebra opened the possibility of algebraically modelling a geometric construction made in GeoGebra and manipulating it with computer algebra algorithms, thus allowing the development of mathematically rigorous and high-performance Automatic Reasoning Tools (ART) to deal with geometric statements ([3]). Hence, the GeoGebra Discovery version of GeoGebra includes some advanced tools offering the user a rich variety of ART to experiment, discover and assert in geometric contexts. In this presentation, we deal with the performance of one of these tools, the LocusEquation command ([1, 2]). The purpose of this command is to automatically discover how to modify a given figure so that an incorrect or unfinished statement becomes true, returning where to place some point in a construction so that a given property is satisfied. We study some constructions where the use of this command gives interesting and unexpected results. As an example of this, we consider the problem of finding the geometric locus of a point C such that the Euler line of the triangle ABC is perpendicular to the side AB. A theoretic analysis suggests that this locus should be the perpendicular bisector of the segment AB, and hence, a line. However, the output given by the LocusEquation command is not a principal ideal, and its zero set includes some additional points which correspond to degeneracy conditions of the construction. We propose a protocol for dealing with this kind of cases, using an external symbolic computation software to show how this protocol works, until these methods are implemented in Giac and in GeoGebra Discovery.

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The module of relations of a degree 4 polynomial

Álvaro Serrano Holgado, Universidad de Salamanca

The question of the triviality of the module of relations of a polynomial (that is, wether or not its roots satisfy nontrivial multiplicative relations) has been studied with some depth, and several sufficient conditions for its triviality exist in the literature, but there is no characterisation yet. In this talk we will explore and completely characterise the case of a degree 4 polynomial, which is the first in which nontrivial relations can occur.

The Nilradical of an Evolution Algebra

Andrés Pérez-Rodríguez, Universidade de Santiago de Compostela

Manuel Ladra, Universidade de Santiago de Compostela

In the context of Lie and Leibniz algebras, the sum of any two nilpotent ideals is also nilpotent by [1, Corollary 4]. Consequently, their largest nilpotent ideal, which is called *nilradical*, is well-defined. Nevertheless, this is not necessarily true for evolution algebras in general.

An *evolution algebra* is an algebra provided with a basis $B = \{e_i : i \in I\}$, called *natural basis*, such that $e_i e_j = 0$ when $i \neq j$. Indeed, evolution algebras are a new type of commutative but non-associative algebras introduced by J. P. Tian in 2008 in [2] that arise with the purpose of modelling non-Mendelian genetics. Hence, the main objective of this talk is to stablish an alternative definition for the nilradical in this context. At the beginning, we will show that the nilradical in the particular case of solvable evolution algebras with 1-dimensional derived subalgebra is well-defined. After been characterised, it will be used to define what we have called the *supersolvable nilradical* of an evolution algebra. Subsequently, we will describe the construction of this new ideal and present some properties which support such definition. Finally, inspired by [3], we will comment on how this supersolvable nilradical may help us when studying Frattini theory in the context of evolution algebras.

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Finite potent endomorphisms and generalized inverses

Diego Alba Alonso, Universidad de Salamanca

Finite potent endomorphisms were introduced by John Tate as a basic tool for his elegant definition of Abstract Residue. However, they have turned out to be an interesting tool to study problems in the framework of infinite dimensional vector spaces. After a brief introduction in the topic of generalized inverses, we will study the map from the set of finite potent endomorphisms over an arbitrary vector space to its power set, consisting on assigning the set of finite potent 1-inverses to a given finite potent endomorphism and its applications.

Shub's conjecture

Alejandro Moreno Becerra, Universidad Complutense de Madrid

The Shub's conjecture is a problem in topological dynamics that concerns estimating the growth rate of the number of periodic points of an endomorphism between manifolds. In particular, let $f : S^n \longrightarrow S^n$ be a $\mathcal{C}^1$ map. The conjecture states that

$$\limsup_{n\to\infty} \frac{1}{n} \log(\#\mathrm{Fix}\ (f^n)) \geq \log|\deg(f)|\,.$$

It was first proposed by Shub in [1], and brought to light again in the Proceedings of the ICM in 2006 [2]. The results regarding the conjecture are scarce and partial: for instance, specific cases in which f preserves a type of foliation on the two-dimensional sphere have been solved [3].

In the talk, we will try to present this conjecture, see some interesting examples, and provide an idea of the mathematical context in which it is framed and the tools available to address it.

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The Conley Index for flows

Bruno Valverde Morales, Universidad Complutense de Madrid

The Conley index is a homotopic index, introduced by Charles Conley as a generalization of the index of a Morse function at a non-degenerate critical point, which contains local information near an isolated invariant set in a dynamical system. In this talk, we will introduce this index and use it to demonstrate some relations between the local dynamics near certain isolated invariant sets and the topology of the phase space. Among others, we will pay special attention to the Conley-Morse equalities, which generalize the classical Morse inequalities. Time permitting, we will apply the Conley-Morse equalities to the study of the Lorenz attractor.

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Synthetic Homotopy Theory in Homotopy Type Theory

Javier Villar Ortega, Universidad de La Rioja, Universidad de Granada

Homotopy Type Theory is a recent field of study that tries to provide a unified framework for developing the relations between Abstract Homotopy Theory, Constructive Logic and Computation. This is done by providing ways of interpreting Constructive Intensional Martin-Löf Type Theory as the internal logic of certain higher categories, that can then be used as models for Homotopy Theory.

This manifests as a novel synthetic way of studying Homotopy Theory, with two-fold objectives. On the one hand, providing new tools for Algebraic Topology (by using these models to verify known results by means of computer assistance, such as with the Blakers-Massey Theorem; or even to develop new arguments, hopefully compatible with the classical setting). On the other hand, using tools from Algebraic Topology for purely Type-Theoretical and Higher-Categorical purposes, such as providing a closed definition for $(\infty, 1) - topoi$, the class of higher categories that provide semantics for these theories; or developing an internal theory of univalent categories.

In this talk, we will present the basic aspects of Univalent Mathematics, develop its more evident connections with classical Homotopy Theory, and hopefully present avenues for further research on the topic, such as the quest for strictification results for the internalization of categories and type theories within these models.

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Alexander Polynomial and *Knot Floer* Homology using grid diagrams

Pablo Regalado García, University of Seville

Knot Theory is a branch of Mathematics devoted to the study of mathematical knots and links. Its origin is related to the classification of links, where link invariants were introduced as a tool allowing to distinguish different links.

This talk focuses in two link invariants studied in my master's thesis: Alexander polynomial and its categorification, *knot Floer* homology. Using grid diagrams, we present a combinatorial approach to both invariants (known as grid polynomial and grid homology).

We also describe the relation between both invariants (grid homology *categorifies* grid polynomial) and their main properties, showing some examples with figures and explicit computations, to illustrate the aforementioned concepts.

The p-adic Jaynes-Cummings model

Luis Crespo, Universidad de Cantabria

Álvaro Pelayo, Universidad Complutense de Madrid

Symplectic geometry is concerned with the study of symplectic manifolds, that is, smooth manifolds M endowed with a closed non-degenerate 2-form. Usually in symplectic geometry one works with smooth manifolds and differential forms over the real numbers. In this contribution we will replace the field of real numbers $\mathbb{R}$ by the field of p-adic numbers $\mathbb{Q}_p$, where p is a fixed prime number, as proposed in [3], and we work out the p-adic analog of the *Jaynes-Cummings model* [2], also known in the mathematics community as the *coupled spin-oscillator*.

The p-adic Jaynes-Cummings model is given by

$$J(x,y,z,u,v) = \frac{u^2+v^2}{2} + z; \qquad H(x,y,z,u,v) = \frac{ux+vy}{2},$$

where $(x,y,z) \in \mathrm{S}_p^2$ and $(u,v) \in (\mathbb{Q}_p)^2$. This is a p-adic analytic integrable system, that is, $\{J,H\} = 0$. Unlike the real case, where the image is described in [4], the image of (J,H) is the whole $(\mathbb{Q}_p)^2$, except if $p=2$, where the image is much more complicated to describe.

The system has two rank 0 critical points, $\{(0,0,-1,0,0),(0,0,1,0,0)\}$, and the rank 1 points have the same description as in the real case. The regular fibers are still 2-manifolds, but not homeomorphic to tori. The fibers of the rank 1 values are more complex then in the real case, where they are circles. The fiber of $(-1,0)$ depends heavily on p: it may be only the rank 0 point (as in the real case), contain the rank 0 point as isolated, or have a more complex structure. The fiber of $(1,0)$ always has a singularity at the rank 0 point, as in the real case.

A full version of this paper can be found in [1].

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The construction of the p-adic L-function of a modular form

Javier Polo Noche, Universidade de Santiago de Compostela

Óscar Rivero Salgado, Universidade de Santiago de Compostela

p-adic L-functions are a powerful tool in algebraic number theory with several applications in Iwasawa theory, an area of research that has provided us with advances regarding the Birch and Swinnerton-Dyer conjecture (BSD). The aim of this talk is to study the construction of the p-adic L-function attached to a modular form of weight $k+2$. To do this, we introduce the L-function of a modular form and prove the algebraicity of the L-values. Using Stevens' control theorem for overconvergent modular symbols we are able to construct the p-adic L-function in the non-critical case ($v_p(\alpha) < k+1$), which verifies the interpolation property expected. Finally, we introduce the critical ($v_p(\alpha) = k+1$) and the Eisenstein cases.

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On the convex hull of integer points above the hyperbola

David Alcántara, Universidad de Cantabria

Mónica Blanco, Universidad de Cantabria

Francisco Criado, CUNEF Universidad

Given $n \in \mathbb{N}$, we are interested in the vertices of the convex hull of all lattice points $(x, y) \in \mathbb{N}^2$ at or above the hyperbola $xy = n$, that is, points satisfying $xy \geq n$. We show that the number of them is in $\Omega(n^{1/3})$ and $O(n^{1/3} \log n)$. In fact, our bounds apply not only to the standard hyperbola but to arbitrary hyperbolas with asymptotes of rational slope.

We also propose an algorithm that finds the neighbor of a given vertex in said convex hull in $O(\log n)$ time, so that the whole set of vertices can be found in $O(n^{1/3} \log^2 n)$ time. Our motivation is that such an enumeration is in fact a deterministic factorization algorithm for n, although it does not beat the current best known deterministic algorithm for integer factorization, due to Strassen [3], which runs in $O(n^{1/4+\epsilon})$ time. Despite of this, our algorithm can be specialized for semiprimes where the distribution of the ratio between factors is known, and can be easily parallelized, which may make it useful in some cases.

A similar convexity problem was studied by Bárány and Larman [2], who proved that the number of vertices of the convex hull of lattice points within the $(d-1)$-sphere of radius $\sqrt{n}$ is in $\Theta\big(n^{\binom{d}{2}(d+1)}\big)$. The fact that for $d = 2$ this gives $\Theta(n^{1/3})$ may suggest that our lower bound is tight and an upper bound can be improved, but whether this is the case remains unclear.

To prove our lower bound we explicitly find the number of vertices stated. The proof of the upper bound uses a lower bound by Andrews [1] for the volume of strictly convex bodies with many boundary lattice points.

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Solvable and Nilpotent Matroids: Realizability and Irreducible Decomposition of Their Associated Varieties

Emiliano Liwski, KU Leuven

Fatemeh Mohammadi, KU Leuven

We introduce and study the families of solvable and nilpotent matroids, focusing on their realization spaces and their closures. Specifically, we analyze their associated varieties and their irreducible decompositions. Additionally, we derive a finite set of defining equations for matroid varieties related to forest point-line configurations. We also construct polynomials within the ideal of matroid varieties for specific families called weak nilpotent matroids.

Bayesian filtering and the Laplace approximation

Irene Sanz Serrano, Universidad de La Rioja

Most of engineering problems necessitate estimating the behavior of dynamic systems, using data provided a priori. This data is mainly obtained through sensors which may not faithfully collect this information. In this talk we will study filters that can infer the state of these systems from all this noise, while also considering the dynamics of the systems.

We will focus particularly on explaining the particle filter, which is commonly used for systems with strong nonlinearities. Particle filters enable the inference of all state statistics by representing probability densities with a cloud of particles. However, these filters present several challenges, such as the sampling step, as they tend to degenerate the solution to a single particle. This can be solved by using a resampling technique, though it can also present problems. Then, we will discuss an improvement to the filter using the Laplace approximation, which will allow us to take samples of the system's state that are closer to our estimate.

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Linearization and stability in fluid dynamics

Miguel Martínez González, ICMAT - Universidad Autónoma de Madrid

The global existence of solutions is a key topic in fluid dynamics. In this talk, we will present techniques that help analyze the long-time behavior of solutions.

We will also explore some existing results on the stability and instability of solutions, alongside an introduction to the spectral theory of unbounded operators. A $1D$-model will be presented as an example to illustrate this theory.

Basketball vs. Watermelon: From Which One Will The Ant Escape First?

Fernán González-Ibáñez, University of Münster

In 1827, Robert Brown “discovered" the irregular motion of pollen particles suspended in water under a microscope, a phenomenon now known as Brownian motion. Later, in 1905, Albert Einstein brought this concept into the realm of physics, providing a theoretical explanation. However, it was not until 1923 that Norbert Wiener formalized the mathematical model of Brownian motion, known as the Wiener process.

One key property of Brownian motion is stochastic completeness. This property can be characterized in various ways, including the uniqueness in the Cauchy problem for the heat equation within the class of bounded solutions [1]. Subsequent studies have extended this analysis to domains in manifolds [2] and minimal submanifolds [3].

In this talk, we will use the previous works to see the relation between the metric and this property. We will introduce the concept and examining the relationship between stochastic completeness and the torsional rigidity of a domain. The main result of this talk will be the evolution of the torsional rigidity when we apply the inverse mean curvature and the Ricci flow on manifolds. A well known fact of the Berger spheres is the evolution under the Ricci flow. Starting from any element of this family of metrics, naming it as watermelons, it will evolve to a round sphere, also naming it as basketball. Then we will give some bounds on the evolution of the mean exit time over this spheres [4].

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Arnold-Liouville theorem and the geometry of toric regular Lagrangian fibrations

Ignacio Megía Pérez, Universidad de Salamanca

Antonio López Almorox, Universidad de Salamanca

In this presentation I intend to summarise my final degree project, which is dedicated to the study of Hamiltonian systems from a geometric viewpoint.

We start by briefly reviewing the basic notions of *Symplectic Geometry*, which constitutes the mathematical structure under Hamiltonian Mechanics and allows us to define the concept of *Hamiltonian dynamical system.*

Once the necessary language is established, we can introduce the so called *integrable Hamiltonian systems* and enunciate Arnold-Liouville theorem. This result gives a characterisation of the local geometry of such systems and demonstrates the existence of a special set of coordinates known as *action-angle variables.* If there is enough time, we may see a quick overview of how the theorem can be demonstrated, which is long but, in my opinion, very beautiful. After that, we show the algorithm that physicists use to compute action-angle variables and illustrate it with an example that can be graphically visualized.

The subject of the last part of the project is *regular Lagrangian fibrations*, which are essentially a generalisation of integrable Hamiltonian systems. It turns out that these objects have an additional *affine structure* which is the key to understand the relation between different systems of action-angle coordinates. Finally, we can enunciate Duistermaat monodromy theorem, one of the main results to analyse the global existence of action-angle variables, and show an example of its application.

The list of references takes a whole page, so I only include the most important for me.

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An energy formula for fully nonlinear degenerate parabolic equations in one spatial dimension

Ester Beatriz, Universidade de São Paulo

Phillipo Lappicy, Universidad Complutense de Madrid

Energy (or Lyapunov) functions are used to prove stability of equilibria, or to indicate a gradient-like structure of a dynamical system. Matano constructed a Lyapunov function for quasilinear non-degenerate parabolic equations [3]. We modify Matano's method to construct an energy formula for fully nonlinear degenerate parabolic equations:

$$f(x, u, u_x, u_{xx}, u_t) = f(x, u, p, q, r) = 0, \tag{1}$$

where $f \in C^2$, with $f_q \cdot f_r \leqslant 0$, $f_r \neq 0$ and $f_q(x, u, p, 0, 0) \not\equiv 0$.

In particular we compute the energy function for several variations of the Inverse Mean Curvature flow, Mean curvature flow, ρ-Laplacian diffusion, Porous medium equation.

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Dvoretzky's theorem

Julia Sánchez Loscertales, Universidad de Sevilla

Dvoretzky's theorem is an important result in the development of functional analysis, because it broke the idea that certain results in finite dimension n can be translated into the infinite dimensional setting by taking $n \to \infty$. It says that the unit ball of any norm in $\mathbb{R}^n$ has a section with dimension approximately $\log n$, that is almost Euclidean. However, it is not true that any infinite-dimensional Banach space has an infinite-dimensional section almost Euclidean, which is the result that we would get by taking limit in the dimension. In this talk, we will state and prove this result.

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Understanding Topological Quantum Field Theories

Enrique Aycart-Maldonado, Complutense University of Madrid

Topological Quantum Field Theories (TQFTs) are powerful algebro-geometric tools that provide us with a new way of computing topological and algebraic invariants of closed manifolds. Although they have their origin in theoretical physics, TQFTs have generated great interest in the mathematical community, being the subject of significant results at the intersection of algebraic geometry, algebraic topology, and theoretical physics.

In this talk, we shall introduce TQFTs and explore how they can be used to compute invariants of surfaces. Specifically, we will characterize two-dimensional TQFTs in terms of Frobenius algebras. Time permitting, we will show how this method of computing invariants can be applied to the study of character varieties.

Derived categories and the Ext functors

Javier Díaz Cabrera, Universidad de Sevilla

Given an abelian category $\mathcal{A}$, we can consider the category of complexes over $\mathcal{A}$ and do the quotient over the homotopy relation, giving rise to the homotopy category of complexes $\mathbf{K}(\mathcal{A})$. Then, we can localize it with respect to the class of morphisms that induce isomorphism on homology. This new category is called the derived category of $\mathcal{A}$ and is denoted by $\mathbf{D}(\mathcal{A})$. In this talk, we will explore the construction of $\mathbf{D}(\mathcal{A})$ and its relation to the functors $Ext^n(-,-)$, which are fundamental in homological algebra.

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Euler Calculus and Euler Transforms: Applications to Data Analysis and Origami

Jesús Gacías Franco, UNED

In recent years, topological data analysis has proven to be a powerful tool to summarize and encode geometric datasets. The Euler Characteristic Transform (ECT) is one such encoding mechanism [1], whereby any "tame" compact shape $M \subseteq \mathbb{R}^n$ is represented as a function $\mathrm{ECT}(M) : \mathbb{S}^n \times \mathbb{R} \to \mathbb{Z}$. The transform is based entirely on the simple topological invariant that is the Euler characteristic, moreover, its many favorable properties (injectivity, measurable outputs, and stability under certain deformations) rely on Euler calculus, a small field bridging integration theory and topology that dares to ask the question: "What if we tried to use the Euler charateristic as a measure?" [2]. The ECT and its variants have already shown promising results in fields such as medical imaging [3] and machine learning [4], and seem to be knocking on the door of another one: mathematical origami.

This talk –based on the speaker's master's thesis [5]– begins by showing the basics of Euler calculus, as well as glancing at some important results (namely, Fubini's theorem and Shapira's inversion formula). The ECT is then introduced and discussed, along with some of its recent variants and extensions that are suited for different purposes. The latter part of the talk will present the speaker's personal research: an extension of the ECT that transforms origami figures. A narrative approach will be taken, showing the inception of the ideas behind this extension, the breakthroughs, the challenges faced, and the unanswered questions.

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Encoding Simplicial Complexes in Quantum Computers

Pablo Munarriz Senosiain, Universidad de La Rioja

Simplicial complexes are topological structures that, in a certain sense, generalize triangles to higher dimensions [1, 2]. They play a central role in various mathematical disciplines, particularly in Topological Data Analysis (TDA), which is used to study the shape of data.

Quantum computing is a computational paradigm based on the mathematical model of quantum mechanics that, in some aspects, can outperform classical computing [3]. For some years now, the way of using TDA in quantum computers has been under study [4, 5, 6, 7]. For that reason, it is essential to understand how simplicial complexes can be encoded quantumly.

In this talk, I will introduce the basic definitions of simplicial complexes and discuss three different approaches to encoding these structures in quantum computers. This exploration aims to provide a foundation for further research into the integration of TDA techniques with quantum computing.

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Hasse-Schmidt differential ideals and regularity of finite type algebras over a field

Adrián Santos Quiles, Universidad Autónoma de Madrid

Let A be a (commutative and unital) ring. The aim of this talk is to introduce Hasse-Schmidt derivations on A and discuss which ideals of A are invariant under a given set of Hasse-Schmidt derivations (the so-called differential ideals) and the relationship between the regularity of A and the lack of differential ideals (i.e., the differential simplicity of A). By doing this, we intend to recover, in a characteristic-free setting, results about the module of derivations $\mathrm{Der}(A)$ when $\mathbb{Q} \subset A$, that do not stay true in a positive or mixed characteristic setting, by replacing $\mathrm{Der}(A)$ with the set of the Hasse-Schmidt derivations, $\mathrm{HS}(A)$.

We will begin by laying the foundations of the theory of differential operators on rings and that of smooth, unramified and étale ring maps, inasmuch as one is naturally led to consider these notions when tackling questions regarding Hasse-Schmidt derivations. After these preliminaries have been settled, Hasse-Schmidt derivations will be defined and a brief account of their motivation and main properties will be given. As described in [1], a generalization to the Hasse-Schmidt and characteristic-free setting of results of Zariski and Seidenberg about differential ideals for sets of derivations on rings containing the field $\mathbb{Q}$ (cf. [3]) is obtained. Finally, following [4], we will describe the behaviour of differential operators under étale extensions. As a consequence of this, given a field k, the ring of k-differential operators of a smooth k-algebra A is generated as an A-module by elements of the form $\delta_1 \circ ... \circ \delta_r$, where the δ_i are components of Hasse-Schmidt derivations on A. This generalizes a well-known result of Grothendieck that states that the ring of differential operators on a smooth algebra over a characteristic zero field agrees with the ring of differential operators of order ≤ 1. We will also establish the equivalence, for finite type k-algebras A whose maximal ideals are all separable over k (i.e., $A/\mathfrak{m}$ is separable over k for all $\mathfrak{m} \in \mathrm{Specm}(A)$), of regularity and $\mathrm{HS}_k(A)$-differential simplicity. By an example, we will argue that the separability of the maximal ideals of A over k is essential to have the aforementioned result.

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Integrability of Hasse-Schmidt derivations in positive characteristic

Laura García Rastrojo, Universidad de Sevilla

The aim of this talk is to introduce the problem of the integrability of derivations of a commutative k-algebra, where k is a commutative ring. It is well-known that the exponential map solves this problem in characteristic zero. However, this map does not make sense over a ring k of positive characteristic. To define what integrability means in such a case Matsumura used the notion of Hasse-Schmidt derivation. Some major results concerning this problem will be shown.

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Spotting the Difference in Operator Algebras

Jorge Pérez García, UAM/ICMAT

Telling operators algebras apart (ie. finding out if they are isomorphic or not) is quite a difficult task. However, by combinig some group theory and analysis techniques, it is possible to find interesting examples of non-isomorphic C*- and von Neumann algebras.

This talk will be a gentle introduction to the topic, featuring some examples of interest and open problems.

Numerical Range: Applications in C*-Algebra, Hilbert Spaces, and Quantum Information Theory

Antonio Jesús Ortega Ravelo, Universidad Autónoma de Madrid

In this talk, an introduction to the theory of the numerical range and its role within Banach algebras and operator spectral theory will be presented. Toeplitz in 1918 defined the numerical range of a matrix, concept that has been extended to bounded linear operators in Hilbert and Banach spaces, offering powerful tools for both algebraic and geometric analysis.

Special attention is given to two applications of the numerical range in pure mathematics, a characterization of C*-algebras (the Vidav Palmer's theorem) and of the Hilbert spaces (the Berkson's theorems). Additionally, the work extends to Physics, particularly in Quantum Information Theory, where numerical ranges are used to model and solve information retrieval problems.

By bridging both mathematical and physical domains, this research highlights the versatility and impact of the numerical range in analyzing operator behavior and solving practical problems in modern scientific contexts.

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Everything you always wanted to know about Hahn-Banach theorem (but were afraid to prove)

Esteban Martínez Vañó, Universidad de Granada

In this talk we will present injective Banach spaces, its closest relatives and its relation with complementability.

An injective Banach space is a generalization of $\mathbb{R}$ in the sense that they can be substituted by the real numbers in Hahn-Banach theorem and it still remains true. More precisely, a Banach space X is called (isometrically) **injective** if for every pair of normed spaces $Y \subset Z$ and any bounded linear operator $t : Y \to X$, there exists a bounded linear operator $T : Z \to X$ that extends t and has the same norm.

These spaces have a well-known characterization (no spoilers) and it turns out that the injectivity impose quite strong requirements on the geometry of the underlying space. For that reason, one can try to relax the conditions of these generalized version of Hahn-Banach theorem to obtain a wider range of spaces. In this line, one can relax the conditions on the operators we aim to extend, for example restricting to extensions of compact operators, or one can relax the condition on the domain of the operators, for example establishing bounds on the density character of these spaces.

We will briefly present both approaches, their relations with complementability and we will conclude with some interesting open questions and the work we are doing about them.

Unconditional and Schauder decompositions in quasi-Banach spaces

Alejandro Marcos Baños, University of la Rioja

Schauder bases, and more in particular unconditional bases, are a key topic in Functional Analysis due to their useful properties. However, some examples of Banach spaces do not have a Schauder basis but can be studied through similar methods, as shown in [3, Section 6].

In this talk, we will explore the possibility of extending these concepts following two different approaches. On the one hand, we will remove the need for local convexity by also considering quasi-Banach spaces. On the other hand, we will replace the traditional notion of bases with a more general framework of decompositions of spaces as a topologic sum of subspaces.

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On a new notion of norm-attaining operators on Banach spaces

Helena del Río Fernández, IMAG-Universidad de Granada

Gensu Choi, Sunchon National University, Korea

Audrey Fovelle, Universidad de Granada

Mingu Jung, Korea Institute for Advanced Study

Miguel Martín, Universidad de Granada

We present a new notion of norm-attainment in which the maximum is required to be strongly attained with respect to the range space. This is a less restrictive concept than the absolute strong exposition introduced by Bourgain in 1977, although it is also closely related to the Radon-Nikodým Property. Using this new concept, we improve some classical results by Uhl (1976) and Schchermayer (1983) and get some analogous results to those obtained by Bourgain (1977).

Quasicomplemented subspaces in Banach spaces

Miguel Ángel Ruiz Risueño, Universidad de Castilla - La Mancha

Mar Jiménez-Sevilla, Universidad Complutense de Madrid

Sebastián Lajara, Universidad de Castilla - La Mancha

In this talk we review the classical theory about quasicomplemented subspaces of Banach spaces, along with some of its applications to the Geometry of those spaces. In addition, we present several recent results which extend the classical ones in contexts like Banach spaces with weak star separable dual.

Posters

Mathematical Modeling of Dynamics in Microbial Communities

Ana Paredes Vázquez, Misión Biológica de Galicia (MBG-CSIC)

Eva Balsa-Canto, Instituto de Investigaciones Marinas (IIM-CSIC)

Julio R. Banga, Misión Biológica de Galicia (MBG-CSIC)

Microbial communities are complex networks of microorganisms that coexist and interact within shared environments. These communities exhibit highly interdependent dynamics that are challenging to study experimentally. Mathematical models, particularly those based on Ordinary Differential Equations (ODEs), are fundamental to modeling these dynamics, providing insights into microbial interactions and facilitating the development of "*in silico*" experiments that simulate real-world community behavior. However, these models often rely on parameters that are either unknown or difficult to measure experimentally, which introduces uncertainty into the model's accuracy [1].

To address this, we first conduct a Structural Identifiability Analysis to ensure that the model's parameters can be uniquely determined from available experimental data. For this purpose, we employ advanced tools such as the GenSSI2.0 Matlab toolbox, and the Julia packages SIAN and StructuralIdentifiability.jl [2]. Ensuring structural identifiability is critical to validating the model's ability to represent the biological system accurately. Once the model is confirmed to be structurally identifiable, we proceed with a Practical Identifiability Analysis with the Matlab toolbox AMIGO2 [3]. This involves using advanced parameter estimation techniques that combine maximum likelihood estimation with global optimization methods.

Overall, this approach bridges the gap between theoretical models and experimental data. The proposed methodology not only advances our understanding of microbial interactions but also offers a pathway for applying these insights to real-world challenges in health, biotechnology, and environmental management.

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The Calderón Problem

Antonio Jesús Ortega Ravelo, Universidad Autónoma de Madrid

In this poster, we introduce the Calderón problem, a fundamental question in inverse problems and partial differential equations, originally posed by A.P. Calderón. This problem, which forms the foundation of Electrical Impedance Tomography (EIT), seeks to determine the internal conductivity of a medium based on boundary measurements.

We will explore classical questions surrounding the Calderón problem, such as the stability and uniqueness of conductivity determination, along with some approaches to reconstruction. These ideas have far-reaching implications, particularly in non-invasive imaging techniques, bridging mathematical theory with real-world applications in areas like medical imaging and geophysics.

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Visualizing Schemes: A Journey Through Examples

Federico Cabrera Linares, Universidad de Granada

Isabel María Moreno Cuadrado, Universidad Complutense de Madrid

This poster presents an intuitive and visual exploration of schemes, inspired by the second chapter of *The Geometry of Schemes* by Eisenbud and Harris [1]. The concept of schemes, a central object in modern algebraic geometry, provides a powerful framework that generalizes varieties. However, schemes often seem abstract and inaccessible to newcomers. Through a series of detailed examples, this poster seeks to demystify schemes by illustrating their concrete manifestations and applications, focusing on affine schemes since virtually all of the differences between the theory of schemes and the theory of abstract varieties are encountered in the affine case.

We begin by introducing the essential concepts required to understand the definition of a scheme. Afterward, the examples start with reduced schemes over an algebraically closed field, which extend the classical notion of affine varieties over such fields. We then progressively relax assumptions, first exploring cases where the ground field is not algebraically closed, and later examining schemes that are not reduced. Finally, we consider "arithmetic" schemes - those that are not defined over any field.

Through illustrations, attendees will gain insight into how schemes serve as a unifying framework, visualizing the underlying geometric properties that give them their unique power in algebraic geometry.

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$\mathbb{Q}$-proper Optimal Linear Fusion Estimation Algorithm for Multiple Sensor Systems with Packet Dropouts.

Felicidad Gutiérrez Trujillo, University of Jaén

José Domingo Jiménez López, University of Jaén

Rosa María Fernández Alcalá, University of Jaén

Jesús Navarro Moreno, University of Jaén

Juan Carlos Ruiz Molina, University of Jaén

This paper analyzes the centralized fusion linear estimation problem in quaternion state-space systems with multi-sensor observations affected by packet dropouts. The packet dropouts are modeled by Bernoulli-distributed independent random variables with known dropout probabilities, as in previous works on estimation using multi-sensor systems with packet dropouts [1, 2, 3]. The problem is addressed in the quaternion domain under properness conditions ($\mathbb{Q}$-properness), which results in a reduced problem dimension and, consequently, lower computational costs [4]. The proposed methodology is based on the innovations approach and Kalman filter techniques. The simulated examples show a computational advantage in the algorithm proposed that allows the optimal estimator to be calculated more quickly. *Grant PID2021-124486NB-I00 funded by MICIU/AEI/ 10.13039/501100011033 and ERDF/EU. Grant PRE2022-101413 funded by MCIN/AEI/10.13039/501100011033 and ESF+.*

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About the reduction of singularities and the existence of separatrices of foliations in $(\mathbb{C}^2, 0)$

Francisco Matanza Sota, University of Cantabria

The main objective of this poster, based in my master's thesis, is to provide a demonstration of two basic results in the study of holomorphic foliations in $(\mathbb{C}^2, 0)$: the result of reduction of singularities after a finite number of blow-ups, and the separatrix existence theorem.

We follow the proof given by Mattei and Mossu in [4] for the result about the reduction of singularities of foliations in a finite number of blow-ups, and analyze the reduction of singularities of a particular type of cuspidal foliation presented by Loray in [3].

For the proof of the separatrix existence theorem, instead of following the original one given by C. Camacho and P. Sad in [1], which is based on the result of reduction of singularities of foliations and the properties of the of the Camacho-Sad index, we stick to the one developed by J. Cano in [2], in which those characteristics of the index are abstracted into another property, simplifying the proof.

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The rolling n-gons

Ignacio Megía Pérez, Universidad de Salamanca

If you are a curious person and have much free time, you may have asked yourself these questions: Why are wheels round? Is the circle the only shape that is able to roll? Could I become rich by selling square-wheeled bikes? Stop loosing sleep thinking about this life-and-death subject.

A *roulette* is the trajectory described by a point that maintains a fixed position relative to a wheel when the latter rolls on a road. The most famous example of a roulette is the cycloid. In this poster, I summarise a group project that I presented together with my classmates Enrique Daniel Pérez and Daniel García González during the second year of our degree. This work is a study, in a Differential Geometry approach, of roulette curves. We start by mathematically characterising the notion of "rolling without slipping" and use it to design an algorithm to calculate the parametrisation of a roulette given the couple wheel-road that generates it. This method is not original, but we extended it, translated the idea to a new matrix language and were able to program it in Wolfram Mathematica, which we employed to make all the roulette graphics that you will find on the poster.

After the previous work, we took advantage of our new knowledge about roulettes to analyse this problem: Is it possible to design a curve such that a polygon can roll on it while its center describes a straight line? The answer is yes, and we give an explicit expression for it, as well as use our program to graphically confirm that, indeed, it works! This is the mathematical solution for the questions we formulated before.

I guess this project may not change the world, but I believe that it is a good example of how mathematics can be fun too.

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Behind the Algorithm: The Historical Paper that Shaped Modern Signal Analysis

Isabel María Moreno Cuadrado, Universidad Complutense de Madrid

This poster will provide an in-depth review of the Fast Fourier Transform (FFT) algorithm , focusing on its revolutionary impact on signal processing and computation. The Cooley-Tukey paper, An Algorithm for the Machine Calculation of Complex Fourier Series [1], will serve as the foundation for our discussion, as it introduced the FFT, a groundbreaking method for efficiently calculating the Discrete Fourier Transform (DFT) and its inverse (IDFT). The FFT significantly reduces the computational complexity of the DFT, making it an essential tool in modern applications involving signal analysis and processing.

The presentation will begin with a historical overview. Following this, we will examine the computational complexity of direct DFT computation and delve into the theoretical basis of the FFT, particularly for the one-dimensional case (FFT-1D). This theoretical analysis will highlight how the FFT enhances efficiency over direct DFT methods. Finally, the poster will feature a recursive FFT algorithm for a specific case, demonstrating how the FFT optimizes practical calculations.

This work aims to showcase both the historical significance and modern-day applications of the FFT, bridging the gap between its theoretical underpinnings and its practical, widespread use today.

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Maximum correntropy filtering for time-varying stochastic systems under measurement quantizations

Jiaxing Li, Harbin University of Science and Technology

Raquel Caballero-Águila, University of Jaén

Jun Hu, Harbin University of Science and Technology

During the past few years, filtering problems in networked systems have stirred much research interest due to their wide range of applications. For non-Gaussian noise, common filtering methods include, but are not limited to, particle filtering, quadratic filtering, and maximum correntropy filtering. Among them, the latter is recognized as an efficient estimation tool, due to its robustness to non-Gaussian noise, its ability to handle outliers or missing data, and other advantages over traditional filtering approaches [1]. On the other hand, because of constrained network bandwidth, sensor measured outputs usually need to be quantized to accommodate practical transmission requirements. Particularly, the dynamic quantization mechanism has attracted significant attention when addressing the state estimation problem (see, e.g., [2, 3]). Based on the above discussions, the primary purpose of this research is to design a maximum correntropy filtering algorithm for time-varying stochastic systems under measurement quantizations. According to available measurements, an upper bound of the filtering error covariance is derived by resorting to the matrix theory, and a proper filter gain is constructed in terms of the maximum correntropy principle. Lastly, a numerical simulation is used to illustrate the effectiveness of the designed filtering algorithm.

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The hybrid Kalman filter and the attitude problem

Jon Ander Alonso, Universidad de La Rioja

José María Pérez, Universidad de La Rioja

In this poster, we present the Kalman, extended Kalman, Kalman-Bucy and hybrid Kalman filters from a mathematically rigurous perspective. We show how the hybrid Kalman filter is used to estimate the attitude of a solid body via quaternions.

Integral equations

Flores, M., University of Salamanca

The generalization of the tautochrone problem, solved by Abel in 1823 was one of the first problems to drive the development of integral equations. In this problem, the goal is to deduce the shape of the plane curve $y(x)$ on which a particle of mass m slides, without friction and solely due to gravity, if the descent time $t = T(y_0)$ from the initial position (x_0, y_0) to the origin is known.

The solution to the previous problem [1] involves an equation of the type:

$$\phi(s) = \int_a^b K(s,t)\phi(t)dt + f(s), \tag{1}$$

where f and K are given functions, ϕ is the function to be found, and the variables s and t range over the interval $[a, b]$. The previous expression is an example of an integral equation, as the unknown function appears under the integral sign.

The present work consists of a brief introduction to integral equations, providing a superficial review of their theoretical framework: integral operators. Key concepts will be introduced, such as their definition and classification through various problems where integral equations naturally arise in their solution. Historical examples include the study of the equilibrium of a string under the action of a point force or the reduction of differential equations to integral equations [2].

So far, historical problems involving integral equations have been discussed. However, integral equations remain relevant today. For example, their theory is applied in the field of nuclear and particle physics in the low-energy regime through the N/D method [3, 4].

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Orbifolds and wallpaper groups

Pedro Fidalgo Martínez, Universidad de Valladolid

Orbifolds are a mathematical concept popularized by Thurston [3] that generalize the concept of a manifold. Instead of being locally Euclidean, orbifolds are locally homeomorphic to the quotient of an open set of $\mathbb{R}^n$ by a finite group of automorphisms. This allows these objects to have well-defined singularities.

Following Conway's ideas [2] we can classify the 17 wallpaper groups using orbifolds. To achieve this we will define the orbifold Euler characteristic χ_{orb}, which will be calculated using the orbifold signature and Conway costs. Conway's *magic* Theorem is the key: it states that orbifolds of the form $\mathbb{R}^2/G$, with G being crystallographic, satisfy $\chi_{\text{orb}}(\mathbb{R}^2/G) = 0$. It turns out that there is a one-to-one correspondence between the wallpaper groups and the orbifold signatures with zero χ_{orb}. A good summary on this topic is a paper by Conway [1].

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Workshops

Homomorphic Encryption, a post-quantum security strategy

Rocío Carratalá Sáez, Universitat Jaume I

Homomorphic encryption (HE) is a groundbreaking cryptographic technique that allows computations to be performed on encrypted data without needing to decrypt it first. This means that sensitive information can remain secure while still being processed, which is particularly valuable in where data is sensitive and its privacy needs to be preserved.

The key principle of HE is the fact that data can be encrypted in such a way that operations, such as addition or multiplication, can be carried out on encrypted data, and once decrypted, the result is the same as if the operations had been performed on the plaintext.

Practical applications and current advancements in homomorphic encryption will be addressed, as well as the main limitations and challenges that still prevent HE from being widely used in many real-world applications.

Quantum computation, quantum cryptography, and conic optimization

Mateus Araújo, University of Valladolid

The advent of quantum computation has raised the possibility of an exponential speedup in the solution of certain problems. This opens up valuable applications, such as simulation of quantum systems, but also creates the danger that our most used cryptosystems will be broken, as they are based on the difficulty of factoring integers. Quantum mechanics also provides a solution, though: it makes it possible to design cryptosystems whose security does not depend on the difficulty of computational problem, but on the basic laws of physics. The design of such a cryptosystem depends crucially on estimating the information that a spy could have obtained. In simple cases this can be done analytic, but in general solving this problem is only possible numerically. Recent advances in conic optimization theory have made its solution much easier, thus enabling us to design better cryptosystems.

Giving voice to math anxiety: echoes from educational research

José Mª Marbán, University of Valladolid

The acquisition of adequate levels of mathematical competence by citizens is a fundamental requirement to address as a society the challenges posed by the new century. Facilitating this development, in an inclusive manner, is, in turn, an unavoidable commitment for all those with competencies in education. Success in this task depends largely on becoming aware of the barriers that impede its progress, among which are elements of an affective nature to which, fortunately, more and more attention is being paid. In particular, the results from the last Programme for International Student Assessment (PISA) show how mathematical anxiety correlates negatively with the level of mathematical proficiency in all the educational systems that participated in the study, regardless of the characteristics of their students or their schools. Moreover, the report shows how one more point on the mathematics anxiety index is associated with an average decrease of 18 points, once the appropriate corrections for students' socioeconomic level have been made (OECD, 2023).

The aim of this talk is to contribute to the awareness of the need to address affective factors in mathematics education, and particularly those related to mathematical anxiety. To this end, the main contributions that research in mathematics education has made in recent decades, together with the questions still unanswered and the most relevant challenges to be addressed in the short and medium term, to understand, characterize and properly address mathematical anxiety, will be presented. As an example of ongoing research to achieve such situation will be illustrated through the guidelines and the first results of the project Design of biomarker-based tools from the identification and treatment of mathematics anxiety from an inclusive approach, a project approved in the 2022 Call for Aid to "Knowledge Generation Projects" – Oriented Research and funded by MICIU/AEI /10. 13039/501100011033 and by FEDER, EU.

An approach to research in Mathematics Education: overview of relevant research topics

Matías Arce, University of Valladolid (Spain)

The critical importance of mathematics in the current world makes it essential to democratize mathematical knowledge. In the words of Alan Bishop, the best mathematics education must be offered to as many people as possible.

The application of general pedagogical and psychological theories is insufficient to explain the teaching and learning processes of mathematics. These theories do not start from the specificity that what is being taught and learned is mathematics, and the characteristics of the mathematical objects and processes. Over the last 50 years, Mathematics Education has been developed and consolidated as a discipline, with the aim of investigating the teaching and learning processes of mathematics in a methodical and systematic way, to understand, interpret and improve them.

In this talk, an overview of research topics in Mathematics Education will be presented, taking as a basis the didactic triangle formed by mathematical knowledge, the teacher and the student, and the relationships between them. In recent years, two examples of very relevant topics are the knowledge and professional development of mathematics teachers and the learning of key mathematical processes such as problem solving or reasoning.

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